

# Volatility During the Financial Crisis Through the Lens of High Frequency Data: A Realized GARCH Approach\*

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## Abstract

We study financial volatility during the global financial crisis and use the largest volatility shocks to identify major events during the crisis. Our analysis makes extensive use of high-frequency (HF) financial data to model volatility and, importantly, to determine the timing within the day when the largest volatility shocks occurred. The latter helps us identify the events that can be associated with each of these shocks, and serves to illustrate the benefits of using high-frequency data. Some of the largest volatility shocks coincide, not surprisingly, with the bankruptcy of Lehman Brothers on September 15, 2008 and Congress's failure to pass the Emergency Economic Stabilization Act on September 29, 2008. The day with the largest volatility shock was February 27, 2007 – the date when Freddie Mac announced a stricter policy for underwriting subprime loans and a date that was marked by a crash on the Chinese stock market. However, the intraday HF data shows that the main culprit was a computer glitch in the trading system. The days with the largest drops in volatility can in most cases be related to interventions by governments and central banks.

**Keywords:** Financial Crisis; Volatility; High Frequency Data; Realized GARCH.

**JEL Classification:** C10; C22; C80

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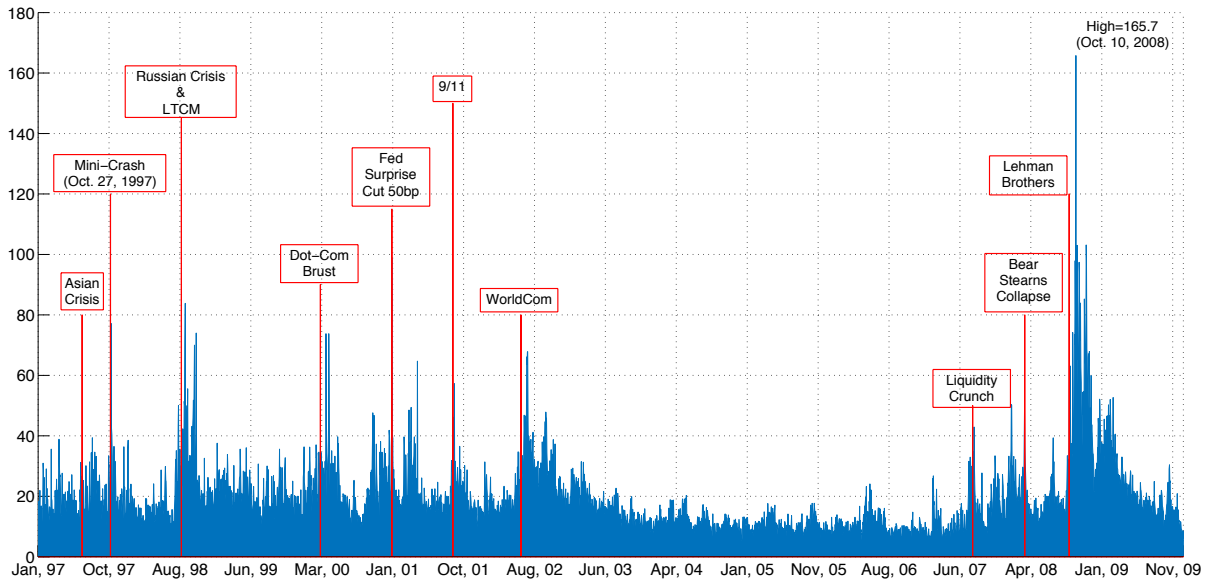
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# 1 Introduction

The aim of this paper is primarily to study financial volatility during the global financial crisis. We use the largest shocks to volatility to identify the major events during the crisis, and utilize high-frequency data to seek out their causes. Our sample spans the period from January 3rd, 1997 to December 31, 2009 that includes several major financial events, which adds perspective to the magnitude of the global financial crisis. High-frequency data are also utilized to construct realized measures of volatility that yields accurate measures of volatility. The relationship between important financial/economic events and our realized measures of volatility is illustrated in Figure 1. The figure presents the annualized realized measure of volatility for the S&P 500 index covering the period 1997-2009. Several important clusters of volatility are observed and associated with major economic events that occurred during this period, including the Asian crisis, the Russian crisis, the Dot-com bubble burst, 9/11, and Lehman Brothers collapse. The highest measured value of volatility was recorded on October 10th, 2008, at 165.7 (annualized).

Figure 1: Realized volatility (1997-2009)



Note: This figure displays the annualized realized volatility for the period 1997-2009 and the time of some of the major crises and events.

Given this context, we first utilize the recently developed Realized GARCH framework Hansen et al. (2012) to extract daily volatilities. This framework utilizes accurate realized measures of volatility that are computed from high-frequency data, that facilitates a measure of daily volatility shocks. Because the Global Financial Crisis was an unusually volatile period, with several unusually large shocks, we propose new variation of the Realized GARCH model

which is less sensitive to outliers. This variant of the model improves the empirical fit during the crisis period. However, the improvements are modest, and it appears that the need for robustification is less important than is the case for conventional GARCH models, see e.g. Harvey (2013, p. 13). This highlights one of the advantages of using realized measures, instead of solely relying on daily returns, as do conventional GARCH models.

Knowledge of financial volatility has considerably increased over the last decade, revolving around two main lines of enquiry: measuring and modeling volatility. This is in part due to the increased availability of high-frequency financial price, which has inspired the development of novel econometric tools that substantially improved the ex-post volatility measurement.

The impetus to the vastly growing literature on measuring volatility came largely from Andersen and Bollerslev (1998), who documented that the realized variance, computed as the sum of squared intraday returns, provides an accurate measurement of daily volatility. The stochastic properties of the realized variance were subsequently studied in Andersen et al. (2001), Barndorff-Nielsen and Shephard (2002), Meddahi (2002), Andersen et al. (2003), Mykland and Zhang (2009). In the meantime, a large number of improved proxies of volatility, which are not sensitive to market microstructure noise were introduced by Zhang et al. (2005), Barndorff-Nielsen et al. (2008), Hansen and Horel (2009), *inter alios*.

The improved measures of volatility motivated the development of volatility models that make uses of realized measures. For instance, Engle and Gallo (2006) proposed the Multiplicative Error Model (MEM) which jointly models returns and realized measures of volatility via a multiple latent volatility processes framework. A simplified MEM structure was subsequently analyzed by Shephard and Sheppard (2010), who refers to their model as the HEAVY model. More recently, Hansen et al. (2012), see also Hansen and Huang (2015) and Hansen et al. (2014), introduced the Realized GARCH model that takes a different approach to the joint modeling of returns and realized volatility measures. The key difference is the presence of a measurement equation that ties the realized measure to the underlying conditional variance.

In this paper we propose and study a new variant of the Realized GARCH model that is sought to be robust to outliers. The new structure is inspired by Harvey (2013) who demonstrated that conventional GARCH models can be severely disrupted by large returns empirically. Harvey (2013) proceeded by proposing a score-driven model that can overcome the problem. By only allowing returns to influence volatility through the score of a  $t$ -distribution, the dynamic is made outlier robust in an intuitive manner. Our robustified Realized GARCH borrows the

outlier dampening feature of the score.

## EMPIRICAL CONTRIBUTIONS...

For a more focused analysis, we zoom in on the events during the recent global crisis (2007-2009) and analyze the days with the largest volatility shocks. We present then the main economic/financial/social/ governmental events that could have induced these shocks. We subsequently use the information in the high-frequency data to identify the exact timing of each shock, which gives us an idea of its real cause. Interestingly, the largest volatility shock is found to coincide with a technical problem in the trading system.

The paper is organized as follows. Section 2 introduces the modeling framework including the robustified Realized GARCH specification. The empirical analysis is presented in Section 3. In Section 4 we discuss the news related to the largest volatility shocks. Section 5 concludes.

## 2 Modeling Framework

### 2.1 Key Variables

We are to study volatility of asset returns,  $r_t$ . In the empirical analysis we use the exchange traded index fund, SPY, to define daily returns because it closely tracks the S&P 500 index and provides us with readily available high-frequency data. The conditional variance of daily returns is denoted by:

$$h_t = \text{var}(r_t | \mathcal{F}_{t-1}), \quad (1)$$

where  $\{\mathcal{F}_t\}$  is a filtration to which  $r_t$  is adapted. Volatility shocks – the key variable in this analysis – are defined by:

$$v_t = \text{E}(\log h_{t+1} | \mathcal{F}_t) - \text{E}(\log h_{t+1} | \mathcal{F}_{t-1}), \quad (2)$$

so that  $100 \times v_t$  is the percentage shock to volatility, induced by news on the  $t^{\text{th}}$  day.

In the rest of this section we detail the econometric modeling of returns and realized measures of volatility, which will lead to our empirical estimates of volatility shocks. After introducing the Realized GARCH framework we detail the robustified version of the model that we introduce in this paper. Readers who are primarily interested in the empirical analysis and less interested in the details of the econometric models can skip the rest of this section and go directly to the empirical analysis in Section 3.

## 2.2 Realized GARCH Framework

The Realized EGARCH model of Hansen and Huang (2015) (with a single realized measure of volatility) is given by the following three equations:

$$r_t = \mu + \sqrt{h_t} z_t, \quad (3)$$

$$\log h_t = \omega + \beta \log h_{t-1} + \tau(z_{t-1}) + \gamma u_{t-1}, \quad (4)$$

$$\log x_t = \xi + \varphi \log h_t + \delta(z_t) + u_t, \quad (5)$$

where  $\tau(z) = \tau_1 z + \tau_2(z^2 - 1)$  and  $\delta(z) = \delta_1 z + \delta_2(z^2 - 1)$ . Here,  $z_t$  and  $u_t$  are typically assumed to be mutually and serially independent and modeled with the specification:  $z_t \sim \text{iid}(0, 1)$  and  $u_t \sim \text{iid}(0, \sigma_u^2)$ .

The three equations are labelled as the *return equation*, the *GARCH equation*, and the *measurement equation*, respectively. The first two form the basis for a GARCH-X model, similar to that estimated by Engle (2002), Barndorff-Nielsen and Shephard (2007), and Visser (2011). The measurement equation is a key characteristic of the Realized GARCH framework, which ties the (ex-post) realized measure,  $x_t$ , to the latent (ex-ante) conditional variance,  $h_t$ . A GARCH-X model is – in isolation – an incomplete description of the data, because it does not model the realized measure. A complete specification of the dynamic properties of both returns and realized measures is achieved by means of the measurement equation. An alternative approach to completing the GARCH-X model that involves additional latent variables was proposed by Engle and Gallo (2006), see also Shephard and Sheppard (2010).

Some of the key features of this model are captured by  $\beta$ , which measures the persistence of volatility, and by  $\tau(z_{t-1}) + \gamma u_{t-1}$ , which estimates the innovation in the conditional volatility. For instance,  $\gamma u_{t-1}$  captures the impact that the realized measure has on the next period conditional variance. The functions  $\tau(z)$  and  $\delta(z)$  are called the leverage functions, as they specify a dependence between returns and volatility that is commonly referred to as the *leverage effect*. Hansen et al. (2012) explored different leverage functions and found a simple quadratic form to be satisfactory in practice. We adopt the same structure in our estimation. In addition, the term  $\tau(z)$  makes reference to the *news impact curve* introduced by Engle and Ng (1993), which shows how positive and negative returns impact expected future volatility.

### 2.3 Robustified Realized GARCH

Several unusually large shocks to returns and volatility occurred during the global financial crisis. Large shocks pose challenges to conventional GARCH models, because these can be highly sensitive to large returns. This motivated Harvey (2013) to suggest a more robust dynamic structure that utilizes the conditional scores of the model. This type of model is known as the dynamic conditional score (DCS) or generalized autoregressive score (GAS) model, see Harvey (2013) and Creal et al. (2012, 2013), respectively.

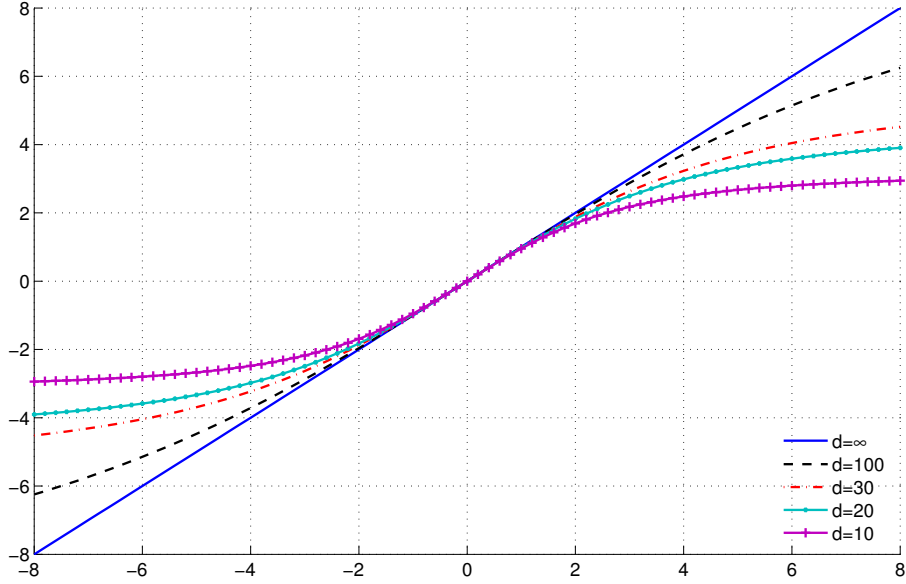


Figure 2: The transformation  $x \mapsto x/\sqrt{1 + x^2/d}$  for various values of  $d$ .

We adopt some insight from Harvey (2013) by introducing parameters that serve to dampen the impact of outliers in returns. For instance, we replace  $z_t$  by  $\tilde{z}_t = z_t/\sqrt{1 + z_t^2/d_z}$  in the GARCH equation, where  $d_z$  is a parameter to be estimated. The transformation is illustrated in Figure 2 for different values of  $d$ . Harvey (2013) deduced the transformation from the score function within a conventional GARCH model, where a univariate time-series of returns are being modeled, see Appendix A for details. In the present context we are modeling both returns and realized measures and both may be affected by outliers. Outliers to returns and outliers in the realized measures, that would translate into unusually large values for  $z_t$  and  $u_t$ , respectively. So we adopt a similar adjustment of  $u_t$ , which measures the shocks to volatility, and substitute  $\tilde{u}_t = u_t/\sqrt{1 + (u_t/\sigma_u)^2/d_u}$  for  $u_t$  in the GARCH equation. Here  $d_u$  is a second robustness parameter to be estimated, analogous to  $d_z$ , and we note that the standard Realized GARCH model emerges in the limit as  $d_z, d_u \rightarrow \infty$ .

The robustified Realized GARCH model has the following structure:

$$r_t = \mu + \sqrt{h_t} z_t, \quad (6)$$

$$\log h_t = \omega + \beta \log h_{t-1} + \tau(\tilde{z}_{t-1}) + \gamma \tilde{u}_{t-1} \quad (7)$$

$$\log x_t = \xi + \varphi \log h_t + \delta(z_t) + u_t, \quad (8)$$

where  $\tilde{z}_t = z_t / \sqrt{1 + z_t^2 / d_z}$  and  $\tilde{u}_t = u_t / \sqrt{1 + (u_t^2 / \sigma_u^2) / d_u}$ , with the leverage functions given by  $\tau(\tilde{z}) = \tau_1 \tilde{z} + \tau_2(\tilde{z}^2 - 1)$  and  $\delta(z) = \delta_1 z + \delta_2(z^2 - 1)$ . Additional variants of the robust model were estimated and compared, see Appendix B for details. In our quasi maximum likelihood estimation we model  $z_t$  and  $u_t$  to be mutually and serially independent, with  $z_t \sim \text{iid}(0, 1)$  and  $u_t \sim \text{iid}(0, \sigma_u^2)$ .

Within the model defined by (6)-(8), the volatility shock,  $v_t = \text{E}(\log h_{t+1} | \mathcal{F}_t) - \text{E}(\log h_{t+1} | \mathcal{F}_{t-1})$ , is given by:

$$v_t = \tau(\tilde{z}_t) + \gamma \tilde{u}_t. \quad (9)$$

So the volatility shock has two components. The first component is the news impact curve that is well known from conventional GARCH models. The second term captures the additional information about future volatility that is embodied in the realized measure. This term illustrates another advantage of using realized measures, as an improved measurement of the volatility shock is made available within the Realized GARCH framework.

## 3 Empirical Analysis

### 3.1 Data Description

We use high-frequency prices for the exchange traded fund, SPY, which closely tracks the S&P 500 index. Our full sample spans the period from January 1, 1997 to December 31, 2009.

We follow the standard practice in the GARCH literature, and model daily close-to-close returns. The realized measure of volatility captures only a fraction of the close-to-close volatility, since high-frequency data is available only from 9:30 am to 4:00 pm every trading day. The realized kernel (RK) is adopted as the realized measure,  $x_t$ , using the Parzen kernel function and a bandwidth that ensures robustness to market microstructure noise. The realized kernel is implemented as in Barndorff-Nielsen et al. (2011) and guarantees a positive estimate, which is useful since we will be specifying our model for the logarithmically transformed volatility. Prior

to computing intraday returns and realized measures, we preprocess the high-frequency data using the cleaning procedures of Barndorff-Nielsen et al. (2009). We also remove unusually quiet trading days (such as days with limited trading hours) around Thanksgiving and Christmas in order to avoid obvious outliers in the realized measures.

In order to quantify the volatilities using an intuitive scale, we will often report the conditional variance and realized measure using an annualized scale. The annualized realized volatility is defined from the realized kernel estimates by:

$$\text{Rvol}_t = \sqrt{250 \times \hat{c} \times \text{RK}_t}, \quad \hat{c} = \frac{\sum_t r_t^2}{\sum_t \text{RK}_t},$$

while the annualized conditional variance (volatility) is defined by  $\text{Cvol}_t = \sqrt{250 \times \hat{h}_t}$ .

### 3.2 Estimation Results

When modeling returns with conventional GARCH models, the specification of the conditional mean typically does not make much difference. This is also true within the Realized GARCH framework. In the present application we have estimated models with constant  $\mu$  as well as models where  $\mu$  is set to zero. The unrestricted estimate of  $\mu$  is small and insignificant, and the resulting time series for  $\hat{h}_t$  are virtually identical whether  $\mu$  is estimated or simply set to zero. The empirical results reported in this paper are for models where  $\mu = 0$  is imposed.

Next we present estimation results for the robustified Realized GARCH model based on daily data for the period January 1, 2006 to December 31, 2009. The numbers in brackets are robust standard errors.<sup>1</sup> We have also estimated the same specification for the full sample period, January 3, 1997 to December 31, 2009, which results in very similar point estimates. These results are presented in Appendix B.

$$\begin{aligned} r_t &= \sqrt{\hat{h}_t} z_t, \\ \log h_t &= \begin{matrix} 0.015 & + & 0.968 \log h_{t-1} & + & 0.377 \tilde{u}_{t-1} & - & 0.179 \tilde{z}_{t-1} & + & 0.054(\tilde{z}_{t-1}^2 - 1), \\ (0.010) & & (0.007) & & (0.054) & & (0.017) & & (0.015) \end{matrix}, \\ \log x_t &= \begin{matrix} -0.530 & + & 1.020 \log h_t & - & 0.130 z_t & + & 0.037(z_t^2 - 1) & + & u_t, \\ (0.082) & & (0.070) & & (0.016) & & (0.008) & & \end{matrix} \end{aligned}$$

with  $\hat{\sigma}_u^2 = 0.154$ ,  $\hat{d}_z = 30.922$ ,  $\hat{d}_u = 18.137$ .

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<sup>1</sup>Robust standard errors are computed using the sandwich estimator, see Bollerslev and Wooldridge (1992).



All key parameters are statistically significant and their signs are meaningful. For instance, the value of the coefficient for  $\tilde{u}_{t-1}$  is  $\hat{\gamma} = 0.377$ , which shows that the realized measure provides an informative signals about future volatility,  $\hat{\beta} = 0.968$  reflects the high persistence in volatility, and  $\hat{\varphi} = 1.020$  suggest that the realized measure is proportional to the conditional variance. This suggests that a fixed proportion of daily volatility occurs during the 6.5 hours the market is open.

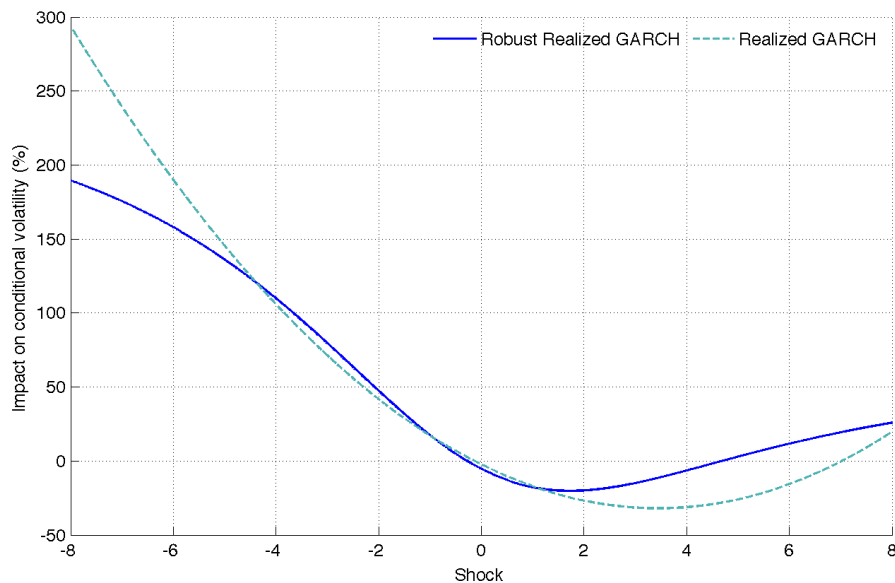


Figure 3: The estimated News Impact Curves based on the Realized GARCH model (dashed) and the robustified Realized GARCH model (solid).

The asymmetric response in volatility to return shocks (leverage effect) is encapsulated in  $\hat{\tau}_1 = -0.179$  and  $\hat{\delta}_1 = -0.130$ . The estimated response in volatility to studentized return shocks,  $z_t$ , is summarized by the news impact curve. The news impact curve is displayed in Figure 3, for both the Robustified Realized GARCH model and the Realized GARCH model. The asymmetric response is pronounced in both models, with negative return shocks have a disproportionately larger impact on volatility than positive return shock of the same magnitude. Figure 3 highlights differences between the robust and non-robust Realized GARCH model, specifically that the former dampens the impact on volatility on days with extreme negative returns shocks.

The time series of the conditional variance,  $h_t$ , implied by the estimated model is presented in Figure 4 along with markers of some of the main events during the Global Financial Crisis. The first spike in volatility was on February 27, 2007, and several other spikes in volatility

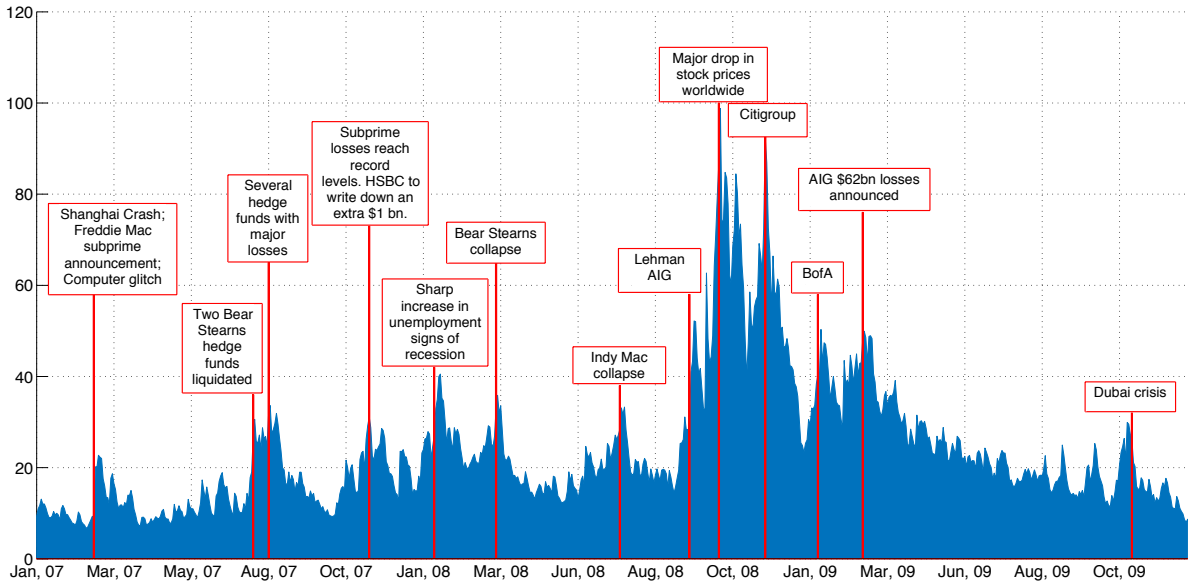


Figure 4: The conditional variance (annualized volatility) estimated with the robustified Realized GARCH model, along with makers of several major events.

are associated with key events such as those related to Bears Stearns, the collapse of Lehman Brothers, and the House of Representatives’ decision to reject the \$700 billion banking-rescue package, etc. We will undertake a closer investigation of the largest volatility spikes in the next section of the paper.

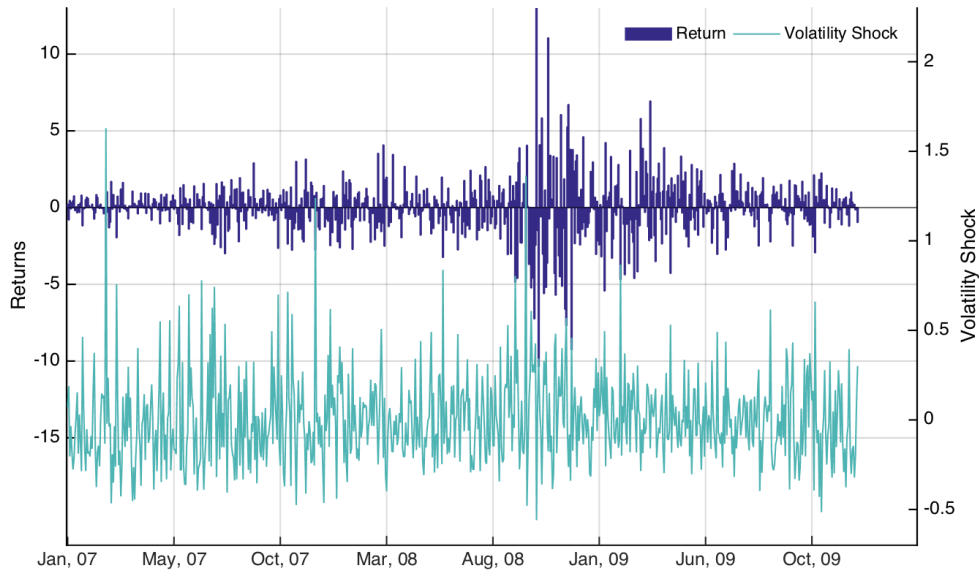


Figure 5: Returns,  $r_t$ , and volatility shocks,  $v_t$ .

The volatility shock,  $v_t = E(\log h_{t+1}|\mathcal{F}_t) - E(\log h_{t+1}|\mathcal{F}_{t-1})$ , summarizes the effect that news

on day  $t$  has on expected future volatility. It can be deduced from the estimated model using (9), and our estimates of  $v_t$  are presented in Figure 5 along with daily returns. As it turns out, the largest estimated volatility shock fell on February 27, 2007. This is partly due to the fact that volatility was relatively low prior to this date (about 9% annualized) so that a 128% increase in expected annualized volatility (which is what  $v_t = 1.629$  translates into) did not bring the volatility to astronomical levels. The non-robust specification has  $v_t = 2.295$  on February 27, 2007, which translates into a 215% increase in annualized volatility.

In Figure 6 we compare the the non-robust Realized GARCH model with the new specification. The upper left panel displays the two series of  $h_t$  along with the realized measure of volatility (using an annualized scale). The two series of  $h_t$  are very similar, occasionally one can see the volatility of the non-robust specification spiking up a bit higher than that of the robust specification. The other three panels displays the same series over 3 week intervals that include the three largest volatility shocks in our sample. Large discrepancies between the volatility series are observed in the upper right panel following the event on February 27, 2007.

A better performance in terms of estimation of the conditional volatility may be associated with the robustified Realized GARCH in the case of the extreme shock that occurred on February 27, 2007. For this specific date, the Realized GARCH appears to overreact to the news, it predicts volatility to be much higher than that suggested by the realized measure the following day. The robust model performs also well as regards the treatment of outliers. It reduces the shock's impact on volatility and drives it quickly to the new level. This makes the robustified Realized GARCH more suited for situations when volatility exhibits "jumps" over a short period of time. Otherwise, the standard and the robust versions of Realized GARCH are comparable in terms of impact, evolution and persistence of shocks (see the panels corresponding to the second and third largest shocks on volatility).

All in all, these results question the trigger mechanism that induced the largest (positive and/or negative) shocks on volatility. In order to provide a general overview, Table 1 summarizes the ten largest positive volatility shocks, as identified by the robustified Realized GARCH, and the major events associated with them. The impact on volatility is calculated as  $100(e^{\frac{1}{2}v_t} - 1)$ , where  $v_t$  accounts for the volatility shock. This impact ranges from 43% to 126%.

Table 2 reports the five largest downwards volatility shocks and lists key events that occurred on these dates. The impact on volatility of downwards shocks tend to be smaller than upwards shocks of the same magnitude, which is known as the leverage effect.

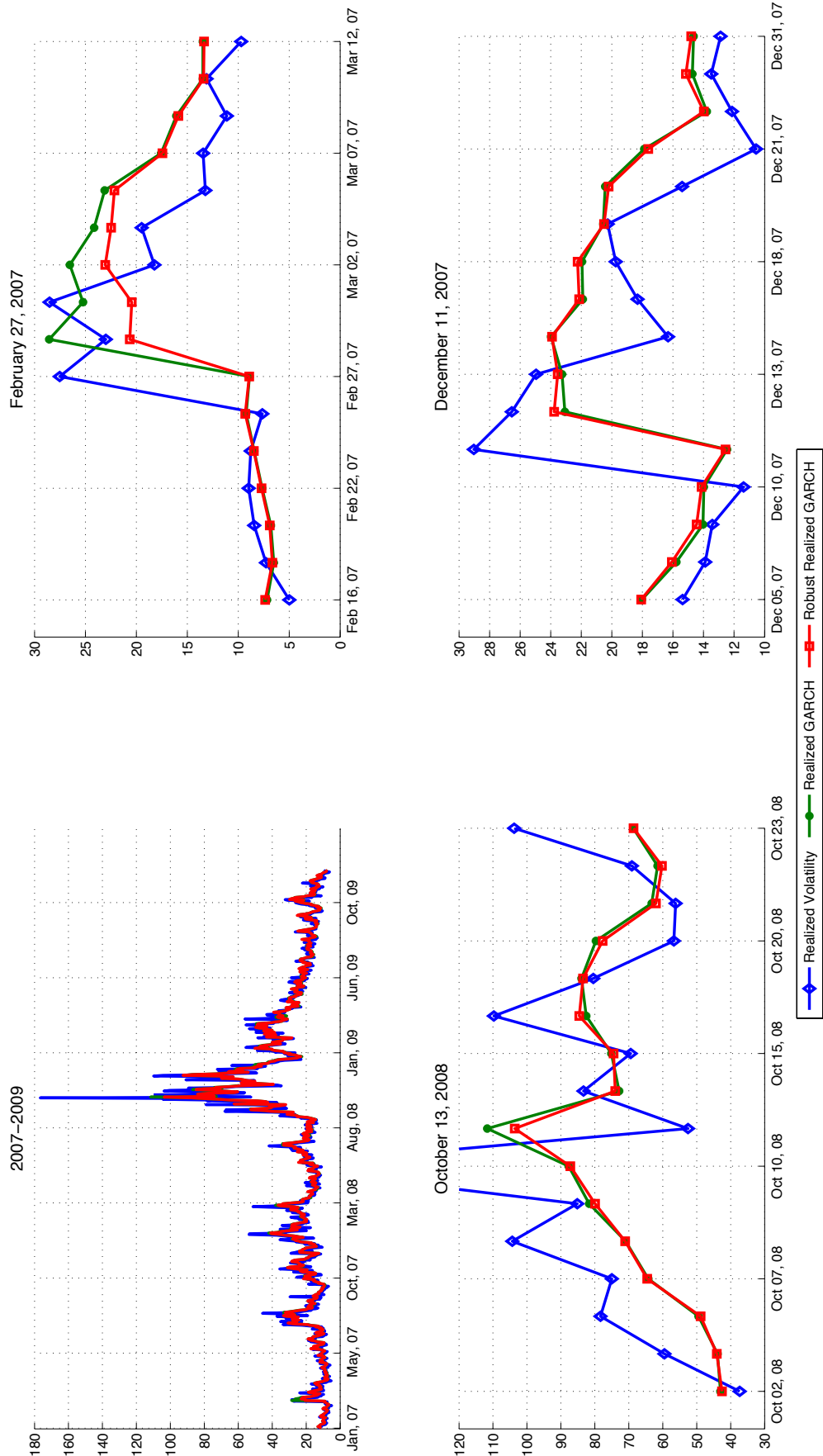


Figure 6: Evolution of the realized measure of volatility and the conditional volatility

Note: This figure presents the evolution of the realized measure of volatility and the conditional volatility for the period of 2007 to 2009, as well as for the periods embedding the three largest positive shocks on volatility, i.e., February 27, 2007, September 29, 2008 and December 11, 2007. The blue line represents the realized measure of volatility, while the red and green lines correspond to the conditional volatility from the Realized GARCH and the robustified Realized GARCH models, respectively.

## 4 News Related to the Largest Volatility Shocks

In this section we will undertake a more detailed study of some of some days in the 2006-2009 sample that we have associated with the largest volatility shocks. Table 1 lists the ten days with the largest positive volatility shocks along with the percentage changes in the S&P 500 and a list of selected news stories. Similarly, Table 2 lists the five dates with the most negative volatility shocks, i.e. the days where news lowered expected volatility by the largest percentage. The percentages volatility shock measures the percentage change in annualized expected volatility, as defined by  $100(e^{\frac{1}{2}v_t} - 1)$ .

For twelve days in the sample (those with the seven largest positive volatility shocks, and five largest negative volatility shocks) we present intraday high-frequency price data along with 13 realized measures of volatility, that are each computed over 30 min intervals. The realized measures are the simple realized variance using 1-minute returns, so that each realized measure is computed from 30 intraday returns. The realized variances are converted in to an annualized volatility scale, by  $RV \mapsto \sqrt{250 \times 13 \times c \times RV}$  where  $c = \sum r_t^2 / \sum x_t$  corrects for the fact that the realized measures only computes volatility over a fraction of the day. For each to the twelve days we will summarize some of the main news and use the high-frequency data to identify the key pieces of news, to the extend this is possible.

### **Tuesday, February 27, 2007 (+128%)**

February 27, 2007 corresponds to the largest volatility shock in our sample, with a volatility shock  $v_t =$  that translates into an expected 128% increase in volatility. On this day, the Dow Jones industrial average fell 416.02 points, which was the largest drop since 9/11, and the S&P 500 and Nasdaq fell by about 3.5% and 3.9%, respectively.

On this date, there where several potentially distressing news stories by the time the (US) markets opened. The Chinese stock market had crashes, there where pessimistic news on the U.S. economy [WHICH?], and the U.S. military base in Afghanistan, which Vice President Dick Cheney was visiting, was attacked by a suicide bomber. Moreover, Freddie Mac announced tighter standards on subprime loans.

The subprime related news story from Freddie Mac is unlikely to have been of major significance to the market turmoil, because the tighter standards were only to be put into effect starting September 1, 2007. The Chinese crash is more likely to have been a contributing factor, as the Shanghai Composite Index had fallen -8.5%, allegedly caused by fears of new regulatory

Table 1: Dates with the ten largest upwards volatility shocks and some key news

Date	Vol. shock	$r_t$ (%)	News
20070227	126%	-3.98	<sup>1</sup> China stock market dropped by 8.8%. <sup>2</sup> Freddie Mac announced tightening standards on subprime loans. <sup>3</sup> NYSE trading interrupted because of a computer glitch around 3:00 pm. <sup>4</sup> News of a suicide bombing at the entrance to the main U.S. military base in Afghanistan during a visit by Dick Cheney.
20080929	98%	-8.16	<sup>5</sup> The House of Representatives rejected the \$700 billion banking-rescue package. <sup>6</sup> Wachovia announced the selling of the banking operation to Citibank. <sup>7</sup> The crisis has spread to the European financial system ( <i>e.g.</i> , the Icelandic government nationalizes the bank Glitnir).
20071211	86%	-2.78	<sup>8</sup> Fed cut the federal funds rate by 0.25% to 4.25%. <sup>9</sup> Large subprime losses announced by Freddie Mac.
20090210	54%	-3.24	<sup>10</sup> Obama administration unveiled the new rescue package but the investment community was concerned that the rescue plan would prove inadequate in the face of a recession. <sup>11</sup> Large layoffs plans are announced by several companies ( <i>i.e.</i> , General Motors, Wal-Mart Stores, UBS).
20080606	52%	-4.69	<sup>12</sup> Unexpected large increase in May, 2008 unemployment rate announced (5.5% up from 5.0% in previous month). <sup>13</sup> Bond guarantors, MBIA and Ambac, were downgraded two notches from AAA to AA. <sup>14</sup> Lehman Brothers announced the plans to raise \$5-6 billion in fresh capital as it disclosed a large second-quarter loss.
20080915	49%	-4.87	<sup>15</sup> Lehman Brothers Holdings Inc filed for Chapter 11 bankruptcy protection. <sup>16</sup> Merrill Lynch acquired by Bank of America.
20070710	48%	-1.43	<sup>17</sup> Standard and Poor's Rating Services put 612 securities on "CreditWatch negative" because of high delinquency and foreclosure rates. Moody's Investors Service downgraded 399 securities and placed an additional 32 securities on review for possible downgrade.
20070313	46%	-1.96	Worries about subprime lending. * The dollar tumbled versus other major currencies.
20071101	45%	-2.37	* Downgrade of Citigroup. * Credit Suisse reported a 31 percent drop in profits. * Exxon Mobil reported a bigger-than-expected drop in quarterly earnings. * Moody's, Standard & Poor's and Fitch put an estimated \$70 billion worth of collateralized debt obligations on review for downgrading. * Economic reports on personal income and spending, manufacturing, foreclosure filings shifted the attention of investors.
20070726	43%	-2.39	* Wells Fargo & Co. announced that it will stop making subprime mortgages through brokers amid escalating late payments and defaults. * NYSE imposed trading curbs to slow down the market in the event of a big move. * Homebuilders posted huge losses (new house sales tumbled 6.6%).

Table 2: Dates with the five largest downwards volatility shocks and selected news.

Date	Vol. shock	$r_t$ (%)	News
20081013	-24%	13.56	<ul style="list-style-type: none"> <li>* Governments to rescue banks through direct capital injections.</li> <li>* The European Central Bank attempts to revive credit market, making unlimited euro funds available (at 3.75% interest).</li> <li>* The U.S. central bank would provide unlimited dollars to the European Central Bank, Bank of England and Swiss National Bank, allowing them to relieve pressures on commercial banks across their regions.</li> </ul>
20091109	-23%	2.25	<ul style="list-style-type: none"> <li>* Finance ministers of the G-20 met over the weekend and pledged to keep the economic stimulus in place.</li> </ul>
20071113	-21%	2.00	<ul style="list-style-type: none"> <li>* Positive statements from CEOs of Goldman Sachs and JP Morgan.</li> <li>* Wal-Mart reported higher than expected third-quarter earnings.</li> <li>* Oil prices falling from near record levels.</li> <li>* Home sales index (for September, 2007) released in the afternoon. PHSI up 0.2% beating expectations of -2.5%.</li> </ul>
20080930	-21%	4.06	<ul style="list-style-type: none"> <li>* The decline in volatility this day is mainly due to the unusually large volatility on the preceding day where a \$700 billion financial bailout plan was rejected by Congress.</li> </ul>
20071221	-21%	1.43	<ul style="list-style-type: none"> <li>* The Federal Reserve announced that it lent \$20 billion to banks at an interest rate of 4.67% in order to support the credit markets.</li> <li>* The “Super SIV” rescue fund was canceled as the consortium claimed that “[it] is not needed at this time”.</li> <li>* Encouraging economic news about personal income and spending.</li> </ul>

measures, such as possible trading taxes. However, this explanation also seems implausible when we turn to the evidence offered by high-frequency data.

Figure 7: Intraday prices and the realized measure of volatility – February 27, 2007

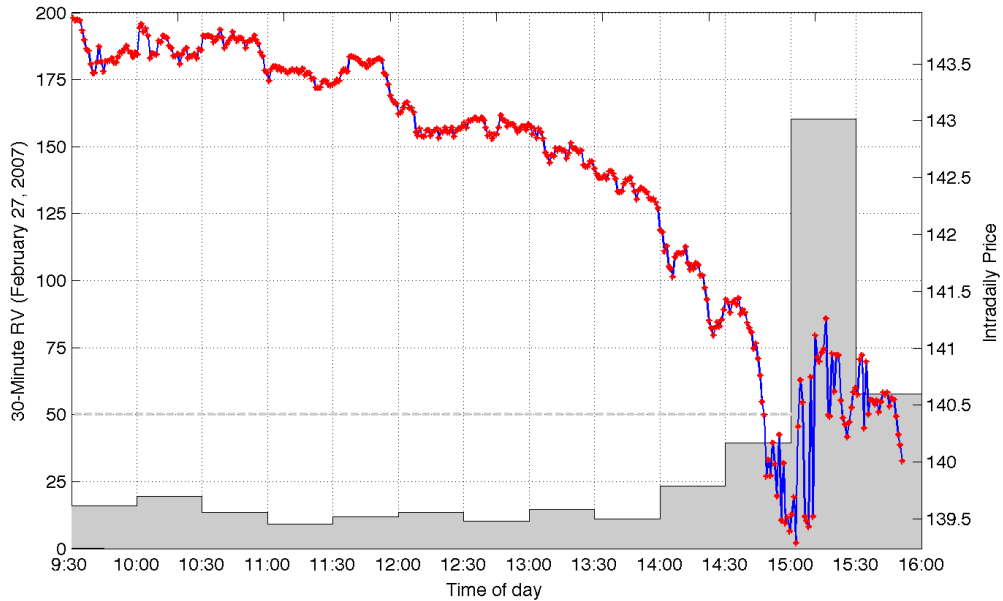
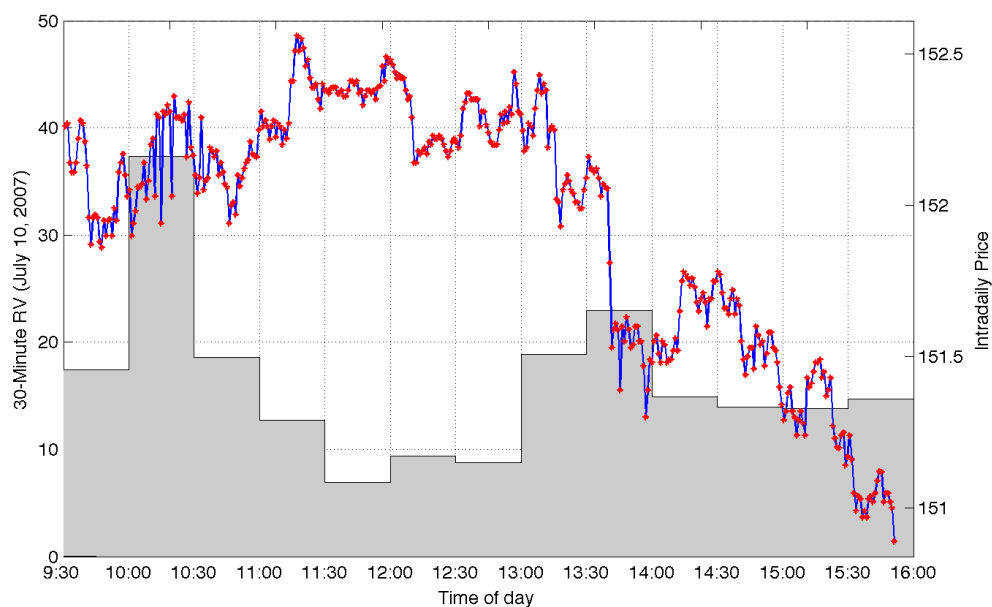


Figure 7 presents the high-frequency prices (minute-by-minute) on the SPY along with realized variances computed over 30 minute intervals. It is evident that markets were not particularly disturbed by the various news stories, including the Chinese crash. What stands out on this day is the increased price fluctuations that begins shortly before 15:00, causing volatility to jump by a factor of eight over a short period of time. This timing coincides with a computer glitch in the trading system. The glitch was that some trades were not reported immediately making resulting in stale prices. According to the Dow Jones spokeswoman: “around 2:00 pm [on that day] the market’s extraordinary heavy trading volume caused a delay in the Dow Jones data systems. [...] and as we identified the problem we decided to switch to a back-up system and the result was a rapid catch-up in the published value of the Dow Jones Industrial Average.” The back-up system was activated around 3:00 pm and at 3:02 pm the index fell by 160 points and continued its depreciation throughout the afternoon. The Dow Jones Industrial average index fell by 546 points in the afternoon. The data for this day provides an excellent example of the valuable information that high-frequency data can offer, and shows that high-frequency data are essential for correctly pinpointing the news events that were the main sources for the market turmoil.

## Tuesday, July 10, 2007 (+48%)

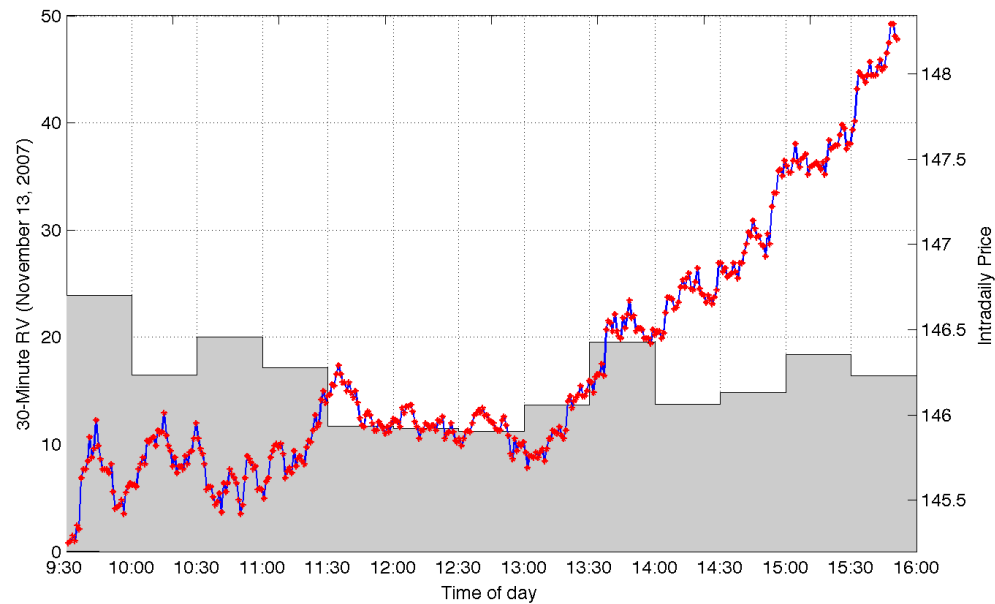
Figure 8: Intraday prices and the realized measure of volatility – July 10, 2007





## Tuesday, November 13, 2007 (-21%)

Figure 9: Intraday prices and the realized measure of volatility – November 13, 2007



On November 13, 2007 the Dow rose by about 320 points. Goldman Sachs and JP Morgan were up 8.5% and 6.2%, respectively, after Goldman Sachs CEO, Lloyd Blankfein, said that the company would not suffer further significant losses related to subprime mortgages, and JP Morgan CEO, Jamie Dimon, had downplayed its exposure to subprime debt. Other good news included Wal-Mart reporting higher than expected third-quarter earnings along with a positive outlook, and oil prices fell (U.S. light crude oil for December delivery fell by \$3.45).

Another, significant news story was a 0.2% increase in the US Pending Home Sales (September, 2007), which was substantially better than the forecast of -2.5% and the -6.5% decline in US Pending Home Sales for the previous month. The release of this story coincided with the afternoon rally in the market on this date.

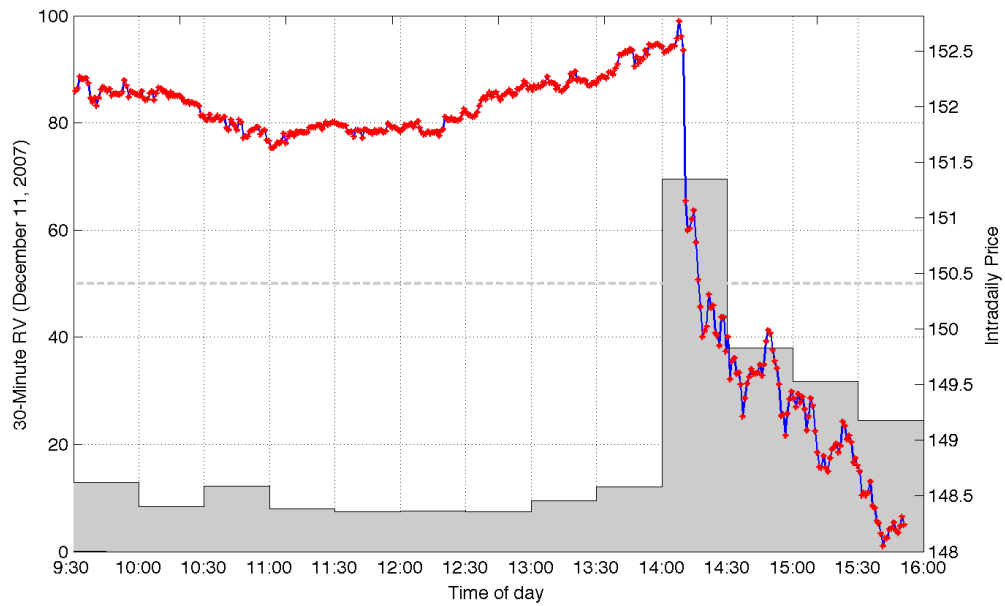
## Tuesday, December 11, 2007 (+86%)

On December 11, 2007, the S&P 500 index fell by 2.5% while the Dow Jones industrial average lost 294 points, or 2.1%, and Nasdaq lost 2.5%. The markets were relatively calm in the morning and the market was up until about 14:15, when it suddenly went into a tailspin while volatility jumped from about 10% to 70% (at an annualized rate). The main news stories of the day were related to the FOMC meeting that resulted in a 25 b.p. reduction of the Fed Funds Rate to

4.25%, which was announced at 14:15. Other news that morning included the CEO of Freddie Mac, Richard Syron, announcing that Freddie Mac would lose an additional \$5.5 billion to \$7.5 billion on top of the \$4.5 billion losses projected previously.

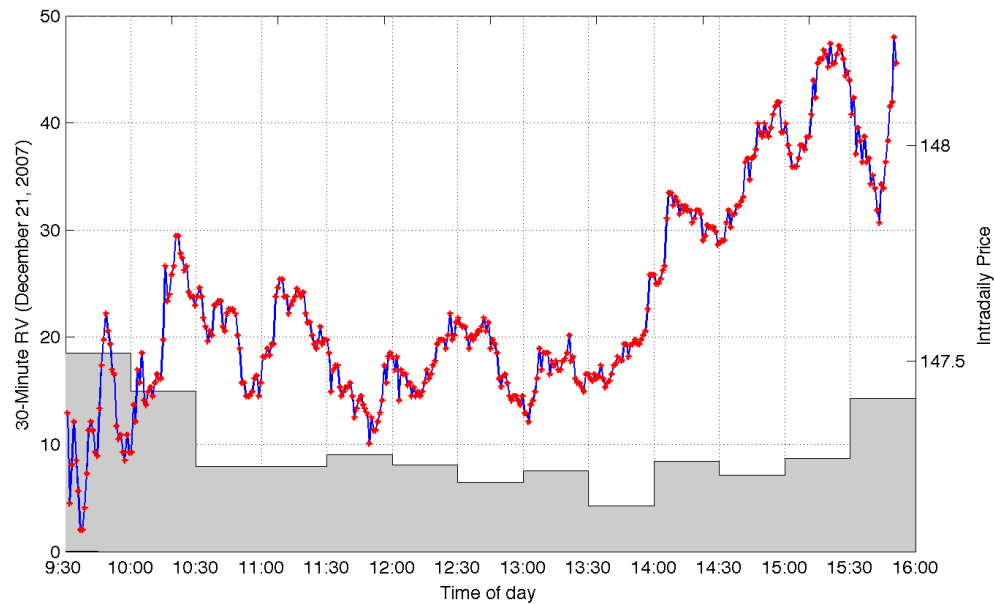
From Figure 10 it is evident that the FOMC announcement triggered the falling prices in the afternoon. The market had expected reduction of the FFR by 50 b.p. and the surprise had an instant market impact that increases volatility for the remainder of the day, see Birru and Figlewski (2010).

Figure 10: Intraday prices and the realized measure of volatility – December 11, 2007



## Friday, December 21, 2007 (-21%)

Figure 11: Intraday prices and the realized measure of volatility – December 21, 2007

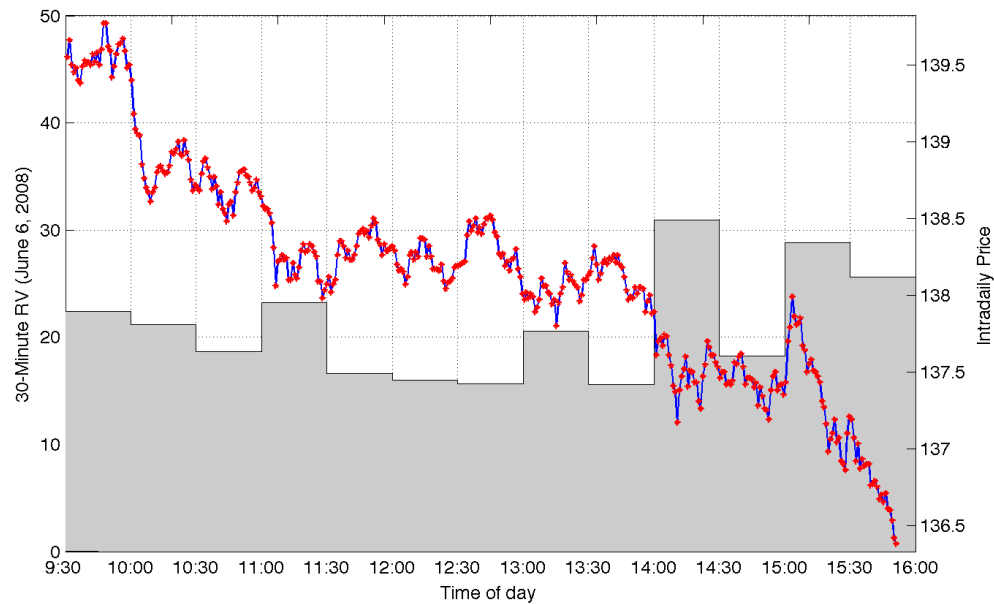


Stocks rose early on December 21, 2007 until the announcement that Merrill Lynch, which was deeply affected by the credit crisis, was in negotiations with Temasek Holdings (a Singapore's state investment firm) to sell a part of Merrill Lynch. In addition, the Wall Street Journal reported impressive earnings from BlackBerry maker Research in Motion. As a consequence, the Dow Jones industrial average gained about 1.2% an hour into the session, S&P 500 index gained 1.3%, and Nasdaq climbed about 1.3%.

In the afternoon on December 21, 2007 it was announced that the plans for a Super SIV (structured investment vehicle) were abandoned. The announcement was followed by the statement that "it is not needed at this time", which the markets may have viewed as good news. The Super SIV, formally named Master Liquidity Enhancement Conduit, was intended to resolve liquidity problems that would otherwise cause fire sales of the SIVs assets. Short term financing was increasingly becoming difficult due to market concerns over the SIVs exposure to subprime mortgages. The consortium behind the Super SIV included major financial institutions, including Citigroup, JPMorgan Chase, Bank of America, Wachovia, and Fidelity. While it was backed by the Treasury Department, former Federal Reserve chief, Alan Greenspan, claimed that the Super SIV would do more harm than good, while other criticized it for being a bailout of banks.

Friday, June 6, 2008 (+52%)

Figure 12: Intraday prices and the realized measure of volatility – June 6, 2008

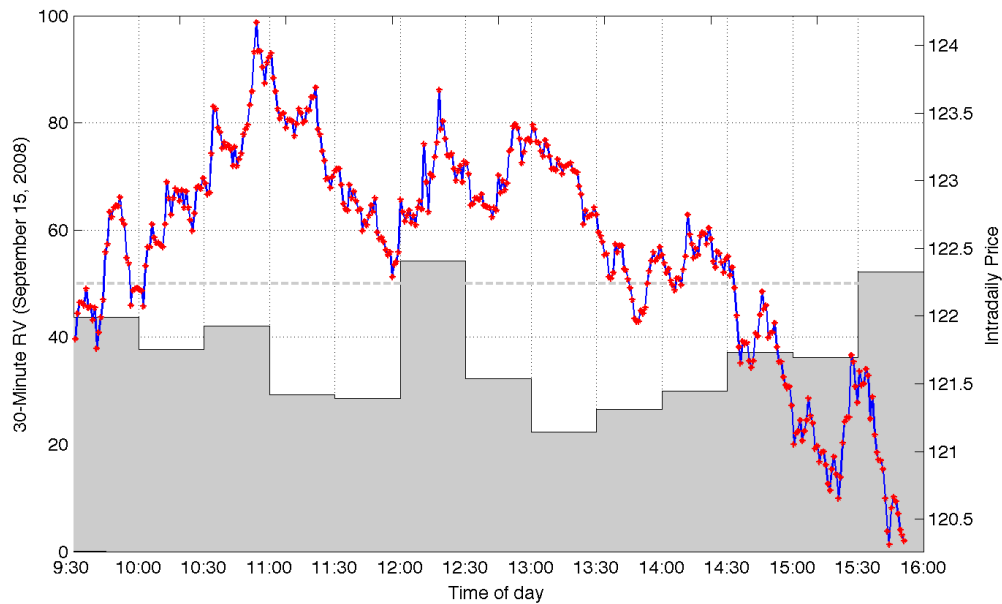


Early in the morning, Dow, Nasdaq and S&P were down after the May jobs report announced the biggest surge in unemployment since 1986. The unemployment rate increased to 5.5% from 5.0% in April, greatly exceeding the expected rise to 5.1%. The jobs report came on the same day that oil prices jumped to \$134 as the dollar lost value against the euro and the yen. It also comes the day after S&P decided to cut the AAA rating of the two largest bond insurers, MBIA (*i.e.*, the world's largest bond insurer) and Ambac (*i.e.*, the second largest insurer). Moreover, S&P warned of further downgrades because of potential further losses from mortgage backed securities. The monolines [??] ratings were downgraded two notches from AAA to AA. The reason behind this action is that further decline of the US mortgage markets and the collateralized debt obligations insured by these actors will lead to more tensions on capital requirements.

On that day, the Dow Jones industrial average lost 395 points, or 3.1%, its biggest one day decline since the start of the subprime mortgage crisis (February, 2007). In addition, the big jump in the unemployment rate and the high oil prices (in the context of a weak labor market outlook and deteriorating housing and credit markets) accentuated the concerns that the U.S. economy was going toward a painful recession.

## Monday, September 15, 2008 (+49%)

Figure 13: Intraday prices and the realized measure of volatility — September 15, 2008



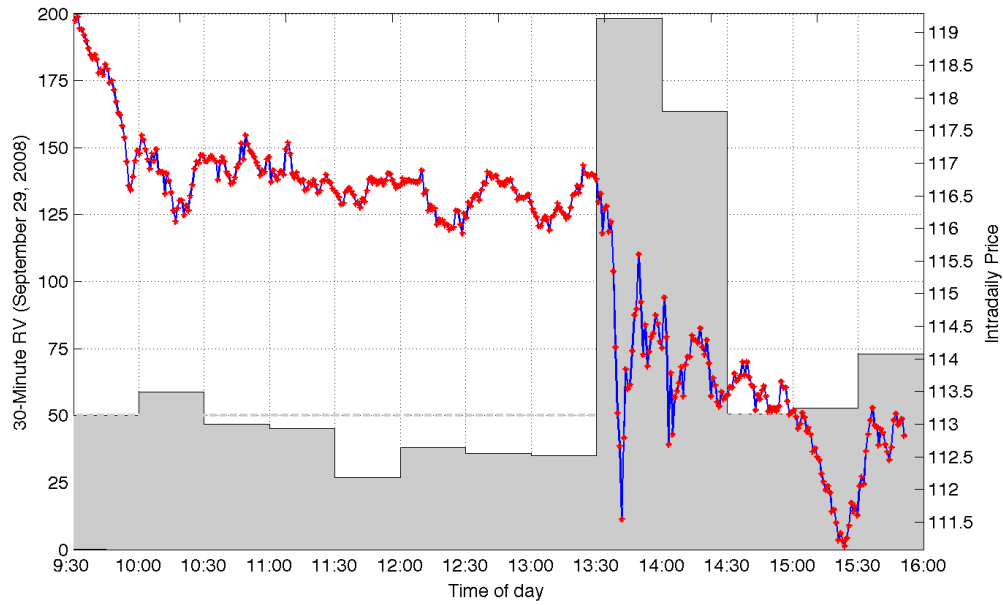
## Monday, September 29, 2008 (+98%)

The second largest shock on volatility occurred on September 29, 2008. As shown in Figure 14, prices plunged significantly in the afternoon between 1:30 pm and 1:45 pm. At that time, the House of Representatives rejected (with a 228-205 vote) the Emergency Economic Stabilization Act of 2008, which triggered a tailspin in the stock market. The amendment was a banking-rescue package that would authorized the Treasury to spend up to \$700 billion for purchasing toxic assets, mainly mortgage-backed securities, and supply cash directly to banks. By the end of the day, the Dow suffered the largest drop in the history of the index, while the Standard & Poor's 500 index was down by 8.8% - its largest drop since the crash of '87.

Other news on September 29, 2008 may also have contributed to the market decline and the spike in volatility. Wachovia announced it was selling its banking operation to Citigroup, and while Wachovia shares lost 81% of their value in the afternoon, Citigroup lost about 12%. Moreover, the British government nationalized the mortgage lender Bradford & Bingley PLC and some European banks collapsed. The German commercial property lender Hypo Real Estate Group opted for a government-facilitated credit line because of difficulties caused by the international credit-market turmoil. The government of Iceland took control of Glitnir, the country's third largest bank, to prevent its collapse. Moreover, over the weekend, Fortis was

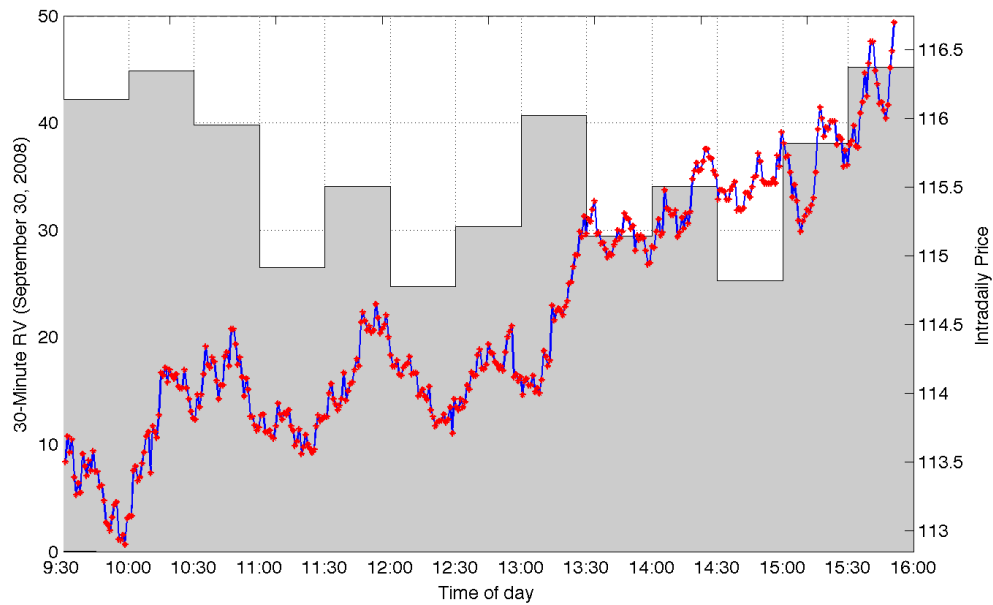
partially nationalized, receiving 11.2 billion capital injection from the Netherlands, Belgium and Luxembourg.

Figure 14: Intraday prices and the realized measure of volatility — September 29, 2008



### Tuesday, September 30, 2008 (-21%)

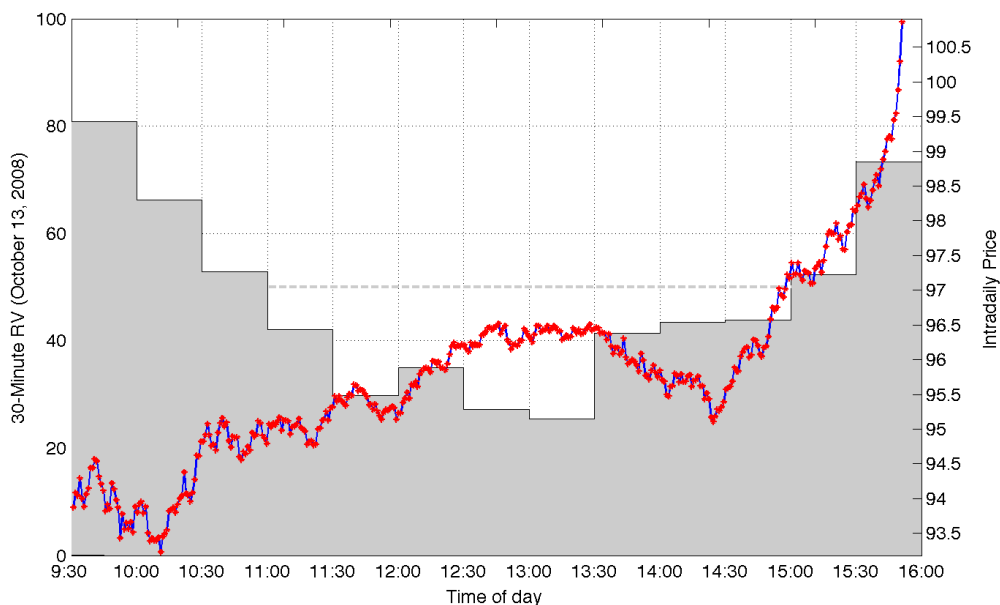
Figure 15: Intraday prices and the realized measure of volatility – September 30, 2008



Stock prices rebounded the day after the Congress failed to pass the government’s \$700 billion rescue plan. The DJIA jumped 485 points, which partially offset the 777 points decline on the previous day. The Standard & Poor’s 500 index and the Nasdaq composite both gained about 5%. Most of the rebound occurred late in the day after the Federal Deposit Insurance Corporation announced expanded deposit insurance by increasing the limits, which was immediately supported by both presidential candidates, Barack Obama and John McCain.

### Monday, October 13, 2008 (-24%)

Figure 16: Intraday prices and the realized measure of volatility – October 13, 2008



[Rebound from October 10?]

Stock markets around the world jumped higher the day after the leaders of 15 European nations gathered in Paris at a first formal meeting, since the launch of the currency back in 1999. Their main goal was to propose measures to combat Europe’s credit crisis. The meeting was organized around four panel discussions on the following themes: i) facilitating the access of banks to capital resources such as to continue the proper financing of the economy; ii) global plans for governments to rescue banks through direct capital injections (*e.g.*, buying soured mortgage assets from banks, injections of capital, etc.); iii) an efficient recapitalization of distressed banks and other appropriate means to support the banking system on the road to recovery; iv) urging regulators to ease the “mark-to-market” accounting requirements based

on the evaluation of assets at their current price. The world's leading nations agreed hence to act together in a comprehensive wide ranging plan to rescue the troubled banking system by adding capital through investment and by guaranteeing inter bank lending.

Shortly before stocks started trading on October 13, 2008, the British Treasury announced the investment of \$63 billion into three major banks (*i.e.*, the Royal Bank of Scotland, HBOS and Lloyds TSB). Investors were also reacting to several other government measures: the Federal Reserve announced it will offer an unlimited amount of dollars to three other central banks (*i.e.*, Bank of England, European Central Bank and the Swiss National Bank) in an unprecedented move to juice short term funding markets and relieve pressures on commercial banks across their regions.

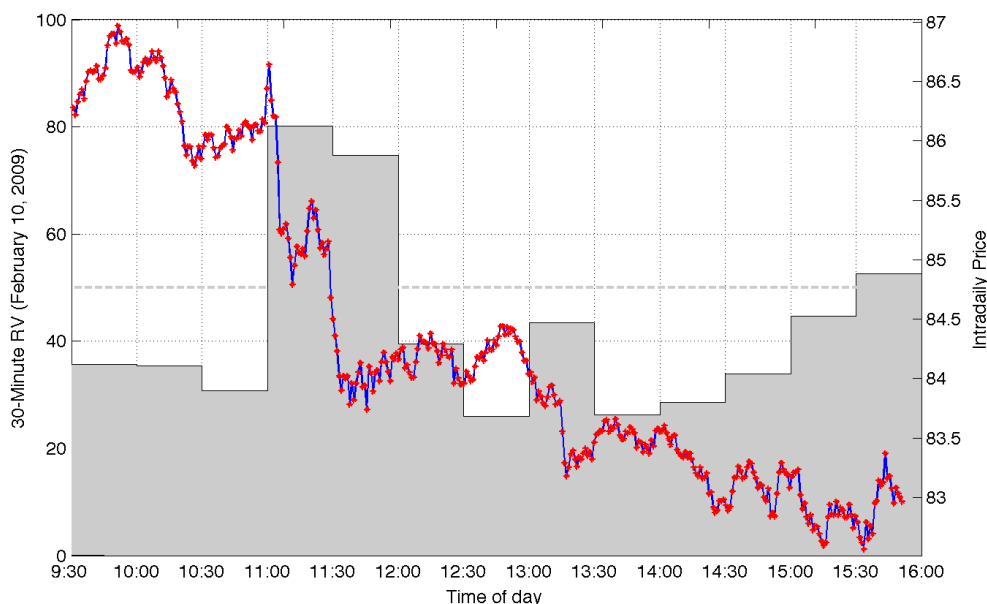
The French president, Nicolas Sarkozy, committed 360 billion in liquidity to French banks, the German government announced a rescue package worth of \$671 billion and the prime minister of Spain, Jose Luis Rodriguez Zapatero, said that Spain will provide up to 100 billion of guarantees for new debt issued by commercial banks in 2008. Moreover, in coordination with other eurozone countries, the Dutch government guarantees interbank lending up to 200 billion. The European Central Bank committed weekly injections of unlimited euro funds at an interest rate of 3.75%.

As a consequence, on October 13, 2008, american stock markets increased and stocks in Europe were trading up (*i.e.*, London's FTSE 100 was up 4.9%, the Cac 40 in Paris gained 6.9% and the Dax in Frankfurt, Germany, was up 8.0%).



**Tuesday, February 10, 2009 (+54%)**

Figure 17: Intraday prices and the realized measure of volatility – February 10, 2009



The day began under an optimistic desired note, as Timothy Geithner, the US Treasury Secretary, announced the introduction of a new Financial Stability Plan in order to replace the original \$700 billion Treasury’s Troubled Asset Relief Program (TARP). The new program was planned in three parts: i) the reinforcement of the stress testing procedures within each banking institution; ii) the development of a new Public-Private Investment Fund, which would provide government capital and government financing helping hence to the recovery of private markets; iii) the revival of the secondary lending markets by a commitment (together with Federal Reserve) up to a a trillion dollars to support a Consumer and Business Lending Initiative.

Nevertheless, the government’s bank rescue plan failed to reassure investors. Stephen Stanley, chief economist at RBS Greenwich Capital categorized the plan announcement as “a huge disappointment”. He added that “there’s been an incredible buildup for weeks and then they release a plan that has little in the way of details.” The Geithner’s late morning speech [WHAT TIME DID HE SPEAK?] degenerated hence in a decrease of the stocks prices, which continued after he finished outlining the plan. The Dow Jones industrial average lost 382 points (or 4.6%) and the loss accelerated in the afternoon up to 422 points. The Standard & Poor’s 500 index lost 43 points, or 4.9%. The Nasdaq composite lost 66 points, or 4.2%.

The bad setup of the day was also fed by large layoffs plans announced by several companies:

*i.e.*, General Motors announced cutting 14% of its salaried jobs [WHAT?] around the world, as well as the salaries of the remaining employees. In order to reduce costs, Wal-Mart Stores said it is also cutting 800 jobs, while UBS presented the layoffs of 2 000 workers and unveiled \$17 billion quarterly loss in the last three months of 2008.

### Monday, November 9, 2009 (-23%)

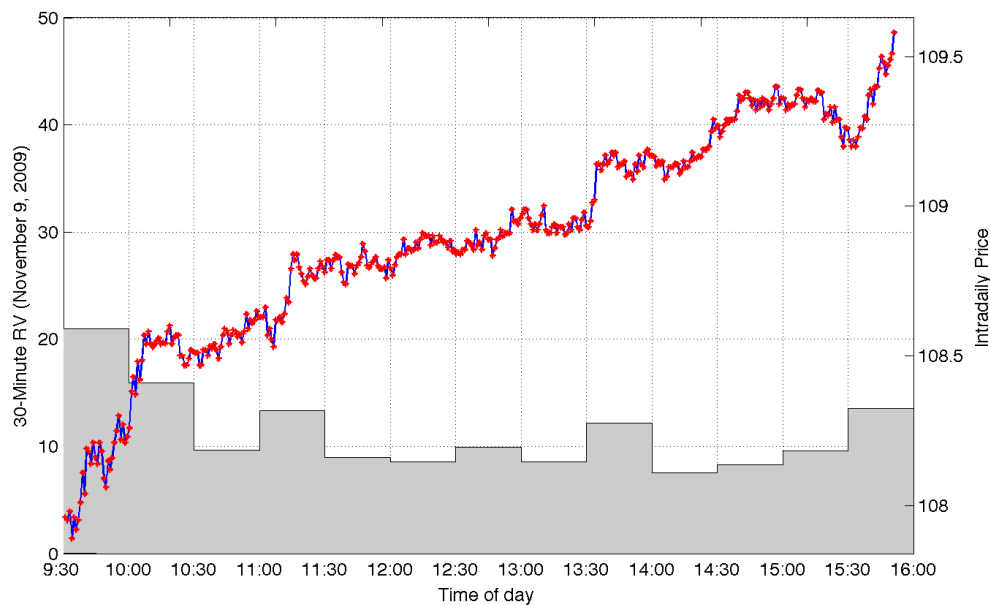


Figure 18: Intradaily prices and the realized measure of volatility – November 9, 2009

On November 9, 2009, stocks prices rose while volatility fell in response to an announcement made by the Group of 20, which met over the weekend and decided to keep economic stimulus in place (*i.e.*, the American Recovery and Reinvestment Act of 2009). This economic stimulus, also known as Obama Stimulus Plan, refers to the \$787 billion plan approved by Congress in February, 2009, and designed to help to the recovery of the economy. Its spending was mainly devoted to tax cuts, unemployment benefits and job creation.

## 5 Conclusion

In this paper we have analyzed volatility during the financial crisis. Using high frequency data and the Realized GARCH model enabled us to identify the days with the largest volatility shocks. A deeper investigation of intraday high-frequency data helped us identify the main culprits

behind these shocks, by comparing the timing of specific events and news announcements with the fluctuations observed in the high frequency data.

As an econometric contributions we propose a new variant of the Realized GARCH model, which is sought to be more robust to outliers. The modification is inspired by Harvey (2013), from whom we adopt a simple transformation that dampen the influence of the outliers on the volatility dynamics. The robustified Realized GARCH improves the empirical fit in terms of the log-likelihood function, but the gains are relatively small, and a rigorous comparison is made difficult by the fact that outliers are rare, so that only a few observations drive the empirical differences between the two specifications.

From the estimated model it is straightforward to extract the volatility shock that measures how much expectations about future volatility increases in response to news on a given day. We zoomed in on days with the largest positive and negative volatility shocks to seek the .....

Second, we define the volatility shock series that incorporates both the new information about volatility changes captured by the returns and the new information captured by the realized measure of volatility. Using the largest positive and negative shocks on volatility we identify the main events (*e.g.*, financial, economic, governmental, social, etc.) that could have induced these shocks. We observe that the largest positive shock on volatility occurred at the beginning of the recent financial crisis, on February 27, 2007, when the markets were not that volatile. From the intraday data we observe a perfect match between the volatility shock and the occurrence of a computer glitch in the trading system (just before 3 pm). We do not have to neglect the importance of the other events that occurred on the same day, such as the decade's biggest drop in the Chinese stock market, or the Freddie Mac announcement about tightening standards on subprime loans. However, their impact would have been overestimated if we had not proceeded with the more detailed intra-daily analysis. The other biggest shocks on volatility were fueled by governmental decisions, economic and financial news or social events.]

Without the detailed insight offered by high-frequency data, other news, such as the Chinese stock market crash or the tighter standards announced by Freddie Mac, may be thought to have caused the volatility shock, or be given more weight than deserved. This association appears to be largely spurious after an examination of the intraday high-frequency data.

## A Motivating the Robustified Structure

The structure of score-driven models, see Creal et al. (2012, 2013) and Harvey (2013), is motivated by the first order conditions that the true parameter values ought to satisfy. Consider the following example where  $y = \sigma z$  with  $z \sim t_d$ , and  $\sigma > 0$  being an unknown scale parameter. If we reparameterize the model with  $\lambda = \log \sigma^2$ , then the log-likelihood function is

$$\ell(\lambda) = -\frac{1}{2}\lambda + c_d - \frac{d+1}{2} \log(1 + e^{-\lambda} \frac{y^2}{d}),$$

where  $c_d = \log[\Gamma(\frac{d+1}{2})/\Gamma(\frac{d+1}{2})/\sqrt{d\pi}]$ . The score is therefore

$$s(\lambda) = -\frac{1}{2} + \frac{d+1}{2} \frac{e^{-\lambda} \frac{y^2}{d}}{1 + e^{-\lambda} \frac{y^2}{d}} = -\frac{1}{2} \left( 1 - \frac{\frac{d+1}{d} z^2}{1 + z^2/d} \right) \simeq \frac{1}{2} (\tilde{z}^2 - 1), \quad \text{with } \tilde{z} = z/\sqrt{1 + z^2/d}.$$

A positive value of  $s(\lambda)$  is a signal that the expected log-likelihood may be improved by increasing the value of  $\lambda$ . Similarly,  $s(\lambda) < 0$  is an indication that a smaller value of  $\lambda$  may improve the objective. In a time series context, with time varying parameters,  $\tilde{z}_t^2 - 1 > 0$  becomes a signal to increase  $\lambda_t = \log \sigma_t^2$ , whereas  $\tilde{z}_t^2 - 1 < 0$  is an indication that  $\lambda_t$  should be lowered. Precisely how much the parameter,  $\lambda_t$ , ought to be changed is less obvious, but a simple starting point is to use a simple autoregressive structure such as  $\lambda_t = \omega + \beta \lambda_{t-1} + \alpha s(y_{t-1})$ . In the robustified Realized GARCH framework we also want to allow for leverage effects, which is the reason we adopt the specification  $\tau(\tilde{z}_t) = \tau_1 \tilde{z}_t + \tau_2 (\tilde{z}_t^2 - 1)$ . This structure, which includes a linear term,  $\tau_1 \tilde{z}_t$ , in addition to the score-motivated term,  $\tau_2 (\tilde{z}_t^2 - 1)$ , is identical to that in Hansen et al. (2012) which the exception that  $\tilde{z}_t$  has replaced  $z_t$ . In our model we maintain the Gaussian distributional specification, and merely use  $\tilde{z} = z/\sqrt{1 + z^2/d}$  to reduce the influence of outliers. A fully-fledged DCS/GAS structure is not needed in order to gain the robustness we seek. Adopting  $t$ -distributions for  $z_t$  and  $u_t$  is relatively straightforward, but would be computationally more cumbersome.

## B Additional Empirical Results

### B.1 Estimated from Large Sample: January 1, 1997 to December 31, 2009.

The empirical results for daily SPY close-to-close returns for the full sample period (January 3, 1997 to December 31, 2009) are:

$$\begin{aligned}
r_t &= \sqrt{h_t} z_t, \\
\log h_t &= \underset{(0.004)}{0.010} + \underset{(0.004)}{0.968} \log h_{t-1} + \underset{(0.037)}{0.325} \tilde{u}_{t-1} - \underset{(0.009)}{0.146} \tilde{z}_{t-1} + \underset{(0.008)}{0.044} (\tilde{z}_{t-1}^2 - 1), \\
\log x_t &= \underset{(0.038)}{-0.414} + \underset{(0.048)}{1.037} \log h_t - \underset{(0.009)}{0.133} z_t + \underset{(0.006)}{0.044} (z_t^2 - 1) + u_t,
\end{aligned}$$

with  $\hat{\sigma}_u^2 = \underset{(0.006)}{0.168}$ ,  $\hat{d}_z = 81.552$ ,  $\hat{d}_u = 6.288$ .

## B.2 Comparison of Different Robust Specifications

We explored a range of specifications in relation to the robustness. All models can be expressed as submodels of:

$$\begin{aligned}
r_t &= \mu + \sqrt{h_t} z_t, \\
\log h_t &= \omega + \beta \log h_{t-1} + \tau(\tilde{z}_{1,t-1}) + \gamma \tilde{u}_{t-1}, \quad \tau(z) = \tau_1 z + \tau_2 (z^2 - 1), \\
\log x_t &= \xi + \varphi \log h_t + \delta(\tilde{z}_{2,t}) + u_t, \quad \delta(z) = \delta_1 z + \delta_2 (z^2 - 1).
\end{aligned}$$

The structure for each of the models is as follows, where M0 is the Realized GARCH model, M5 is the specification used in the paper, and M6 is the most general specification.

- M0:  $z_t = \tilde{z}_{1,t} = \tilde{z}_{2,t}$  and  $u_t = \tilde{u}_t$
- M1:  $z_t = \tilde{z}_{1,t} = \tilde{z}_{2,t}$ , and  $\tilde{u}_t = u_t / \sqrt{1 + (u_t/\sigma_u)^2/d_u}$ .
- M2:  $\tilde{z}_{1,t} = \tilde{z}_{2,t} = \tilde{z}_t$  with  $\tilde{z}_t = z_t / \sqrt{1 + z_t^2/d_z}$  and  $u_t = \tilde{u}_t$ .
- M3:  $\tilde{z}_{1,t} = z_t / \sqrt{1 + z_t^2/d_{1z}}$ ,  $\tilde{z}_{2,t} = z_t / \sqrt{1 + z_t^2/d_{2z}}$ , and  $u_t = \tilde{u}_t$ .
- M4:  $z_t = z_{2,t}$ ,  $\tilde{z}_{1,t} = z_t / \sqrt{1 + z_t^2/d_z}$  and  $u_t = \tilde{u}_t$ .
- M5:  $z_t = z_{2,t}$ ,  $\tilde{z}_{1,t} = z_t / \sqrt{1 + z_t^2/d_z}$  and  $\tilde{u}_t = u_t / \sqrt{1 + (u_t/\sigma_u)^2/d_u}$ .
- M6:  $\tilde{z}_{1,t} = z_t / \sqrt{1 + z_t^2/d_z}$ ,  $\tilde{z}_{2,t} = z_t / \sqrt{1 + z_t^2/d_{2z}}$ , and  $\tilde{u}_t = u_t / \sqrt{1 + (u_t/\sigma_u)^2/d_u}$ .

The empirical results....

As previously noted, the robustified Realized GARCH model controls the impact of jumps on volatility and on the realized measure. Moreover, its general form can contain many simplified specifications. For instance, we can consider a specific form of the robust version that deals only with the impact of jumps on the conditional volatility (*i.e.*,  $\tilde{z}_{2,t} = z_t$ ), or another form that treats identically the impact of jumps in the return series on the conditional volatility and the realized measure (*i.e.*,  $d_{1,z} = d_{2,z}$ ), and so on.

In this section we shed light on the robustified Realized GARCH structure (both general and simplified forms) and subsequently compare its performances in terms of empirical fit with those of the standard Realized GARCH. To this end, we estimate the robust version of the model previously presented, five other different models derived from this robust specification (denoted M1-M5), as well as the standard Realized GARCH, for the period of 2006 to 2009. Table 3 presents the outline of the estimation diagnostics, as well as the value of the log-likelihood function.

The best model in terms of log-likelihood is M6, albeit it is closely followed by M5. However, the difference between these two models is not statistically significant. Moreover, the new parameter of the transformed innovation term that appears into the measurement equation of M6 is quite large, allowing us to presume  $\tilde{z}_{2,t} = z_t$ . It means that we can control for the asymmetry and the impact of the outliers by changing the innovations structure only in the conditional volatility formula (*i.e.*, the GARCH equation). The estimated parameter associated with the number of degrees of freedom appearing in the transformed innovation term  $\tilde{u}_t$  ( $d_u = 18.14$ ) is lower than that associated with  $\tilde{z}_{1,t}$  ( $d_{1,z} = 30.92$ ), which suggests that the influence of the outliers coming from the realized volatility series is even more rigorously controlled. In addition, the log-likelihood for M5 is 6 units greater than the classical Realized GARCH specification, highlighting the statistical benefits of incorporating the score function into the definition of the innovation terms corresponding to the GARCH equation.

Furthermore, results suggest that the largest volatility shocks occur basically on the same dates in all the seven specification models.

Table 4 reports the values of the ten largest positive volatility shocks along with the corresponding dates of occurrence. It may be seen that the cluster of days is systematically the same across all the models, albeit the order of the days slightly differs.<sup>2</sup> Compared to the robust specifications, the shocks modeled by the Realized GARCH are higher. The values become comparable when the intensity of shocks diminishes.

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<sup>2</sup>Since the parameters are sensitive to some extent to the size of the estimation sample, the order of the days characterized by the largest volatility shocks (either positive or negative) could also slightly change with respect to the estimation period. However, the clusters layout is always the same.

Table 3: Parameter estimates for each of the seven model specifications: The Realized GARCH model (M0) and the six robustified models

	M0 Realized GARCH	M1	M2	M3	M4	M5 (Preferred)	M6
$d_{1z}$			63.536	29.488	33.359	30.922	24.698
$d_{2z}$			63.536	290.770			290.787
$d_u$		12.766				18.137	5.891
$h_0$	0.797	0.812	0.782	0.797	0.803	0.813	0.820
$\omega$	0.006	0.007	0.010	0.015	0.014	0.015	0.018
$\beta$	0.972	0.972	0.971	0.968	0.968	0.968	0.969
$\gamma$	0.368	0.402	0.364	0.354	0.351	0.377	0.411
$\tau_1$	-0.171	-0.171	-0.177	-0.180	-0.178	-0.179	-0.183
$\tau_2$	0.025	0.025	0.043	0.056	0.053	0.054	0.059
$\xi$	-0.518	-0.519	-0.516	-0.528	-0.531	-0.530	-0.529
$\varphi$	1.006	1.005	0.994	1.014	1.022	1.020	1.012
$\delta_1$	-0.128	-0.129	-0.133	-0.130	-0.129	-0.130	-0.133
$\delta_2$	0.037	0.036	0.052	0.042	0.038	0.037	0.040
$\sigma_u^2$	0.157	0.157	0.156	0.154	0.155	0.154	0.154
AIC	4026.1	4026.3	4024.9	4020.1	4019.2	4018.4	<b>4018.3</b>
BIC	4080.0	4085.1	4088.6	4083.8	<b>4078.0</b>	4082.1	4086.9
logL	2002.1	2001.2	1999.5	1997.0	1997.6	1996.2	<b>1995.1</b>

Table 4: Ten largest positive volatility shocks

M0		M1		M2		M3		M4		M5		M6	
Date	$v_t$	Date	$v_t$	Date	$v_t$	Date	$v_t$	Date	$v_t$	Date	$v_t$	Date	$v_t$
20070227	2.295	20070227	2.310	20070227	2.201	20070227	1.666	20070227	1.648	20070227	1.629	20070227	1.584
20080929	1.314	20080929	1.281	20080929	1.393	20080929	1.383	20080929	1.388	20080929	1.364	20080929	1.305
20071211	1.213	20071211	1.167	20071211	1.280	20071211	1.271	20071211	1.271	20071211	1.239	20071211	1.187
20080606	0.791	20080606	0.787	20080606	0.845	20090210	0.879	20090210	0.877	20090210	0.868	20090210	0.849
20090210	0.779	20090210	0.762	20090210	0.832	20080606	0.840	20080606	0.841	20080606	0.839	20080606	0.828
20070726	0.731	20080915	0.700	20080915	0.763	20080915	0.804	20080915	0.803	20080915	0.804	20080915	0.801
20080915	0.705	20070710	0.696	20070726	0.759	20070710	0.784	20070710	0.777	20070710	0.779	20070710	0.775
20070710	0.701	20070726	0.687	20070710	0.752	20070726	0.778	20070726	0.776	20070313	0.758	20070313	0.756
20070313	0.662	20070313	0.659	20070313	0.720	20070313	0.757	20070313	0.752	20070726	0.743	20071101	0.724
20071101	0.638	20071101	0.640	20071101	0.686	20071101	0.713	20071101	0.710	20071101	0.716	20070726	0.708

Note: This table presents the ten largest positive volatility shocks computed as  $v_t = \tau(\tilde{z}_t) + \gamma\tilde{u}_t$ , along with the corresponding dates of occurrence.

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# Web Appendix for “paper”

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