How much do households really know about their future income?*

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Abstract

We develop a consumption risk sharing model that can in a systematic and consistent way distinguish between households' perceived income uncertainty and income uncertainty as measured by an *econometrician*. Households receive signals on their future disposable income that can drive a wedge between the two uncertainties. Using U.S. micro data to inform the general equilibrium model, we find support for a systematic uncertainty gap: households know more than econometricians such that their perceived income uncertainty is at least 12 percent lower than the uncertainty estimated by an econometrician that is typically used in risk sharing models. For this uncertainty gap, the model jointly explains three distinct risk sharing measures that are not captured in the standard model without a gap: (i) the cross-sectional variance of consumption, (ii) the covariance of consumption with income growth, and (iii) the income-conditional mean of household consumption.

JEL classification: E21, D31, D52
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1 Introduction

What is households' income uncertainty when they decide about their savings to insure against undesirable fluctuations of their consumption? The answer to this question is of central importance to understand consumption risk sharing; only what households don't know yet constitutes uncertainty they seek to hedge. Typically, households' income uncertainty measures stem from aggregating earnings across household members and income types in the population. As pointed out by Browning, Hansen, and Heckman (1999) and again more recently by Cunha and Heckman (2016), this procedure may however create a disconnect between the uncertainty as assessed by an *econometrician* and income uncertainty as perceived by households. Quantifying households' perceived income uncertainty is however a prerequisite for evaluating the welfare effects of hotly-debated reforms such as changes in the progressivity of the tax system. As our main contribution, we develop a risk-sharing model suitable for policy analysis that can distinguish between households' perceived income uncertainty and income uncertainty as measured by an econometrician in a systematic and consistent way. We find that accounting for households' perceived income uncertainty is key to understand risk sharing of households in the United States.[1]

We consider an environment in which risk averse households seek insurance against idiosyncratic fluctuations of their disposable income. As the new element here, we explicitly extend households' information set by signals that inform households on their income in the next period with certain precision. While the stochastic income process constitutes the income uncertainty as assessed by an econometrician, the joint process of signals and income represents households' income uncertainty. The difference between the two income uncertainties depends on the precision of the signals as households' advance information; the more precise are the signals, the lower is households' forecast error for income growth and the lower is households' perceived income uncertainty. The extension of households' information set is motivated by the observation that households typically have more information than their current income to predict their future earnings. Thus, the signals capture a wide spectrum of information relevant for future changes in disposable income that are already known to households before the actual change occurs. Examples for this type of fore-knowledge are information on future performance bonuses, promotions, demotions or wage cuts, wage rises,

[1] Throughout the paper, we use the terms uncertainty and risk interchangeably, and consider known probabilities of random events which differs from the concept of Knightian uncertainty where the probabilities are unknown.

changes in income taxes and transfer but also changes in marital status; a divorce can be interpreted as a signal for a negative income shock while a marriage signals a positive income shock.

In reality, households can smooth income shocks in a variety of ways, involving progressive taxation, family transfers, informal networks or default. To capture these various insurance possibilities, we employ a general-equilibrium model with endogenous solvency constraints stemming from limited contract enforcement as proposed by Alvarez and Jermann (2000). In this model, households have access to a full set of formal and informal insurance contracts with the drawback that these contract are not enforceable under all circumstances.

As our main novel finding, we discover that households have advance information on their future income such that their income uncertainty is lower than what is typically considered in consumption risk sharing models. Employing U.S. micro data to inform the theoretical model, we find that advance information reduces households' mean-squared forecast error for income by 12 percentage points. This implies a systematic gap between the income uncertainty as perceived by households and the income uncertainty as estimated by an econometrician. Accounting for this gap, the model jointly explains three distinct consumption risk sharing measures that are not captured without advance information: (i) the unconditional variance of households consumption in the cross section, (ii) the covariance of current consumption and income growth and (iii) the income-conditional mean of household consumption.

As our main theoretical result, we show that with limited contract enforcement advance information reduces consumption risk sharing. The rationale for this surprising result is that more precise signals decrease the value of insurance for high-income households which limits opportunities for risk sharing between households. Models with limited contract enforcement but without advance information tend to overstate the degree of household risk sharing. Thus, the decrease in risk sharing resulting from households with advance information improves the fit of the model to the data.

In the quantitative exercise, we characterize cross-sectional long-run distributions of consumption, income and wealth across households with advance information. To do that, we develop a stochastic model with an explicit specification of the joint distribution of income and signals as a methodological contribution that is applicable to a large set of macroeconomic models with a recursive structure. Keeping track of the joint distribution of signals and income also imposes a challenge to the computation of optimal allocations with limited contract enforcement.

Given an uncertainty gap of 12 percent, we analyze the quantitative implications for the income-conditional distribution of consumption. We find that advance information also here substantially improves the fit to the data. In particular, the model (almost) perfectly tracks the income-conditional mean of consumption both low, medium and high income earners. Further, advance information helps to attenuate a non-linearity present in limited contract enforcement models without information but absent in the data. In the absence of information, the limited commitment model implies a variance of consumption conditional on a high income that is equal to zero. With informative signals, the conditional variance is positive, bringing the model closer to the data.

Related literature We are not the first to find that households know more than econometricians about their future earnings.[2] The main differences to existing papers are first that we find a quantitative important role for advance information in general equilibrium. Further, we employ a limited commitment model to explicitly capture both formal and informal insurance arrangements while previous papers focused on a standard incomplete markets model with self insurance.

Most closely related to our paper are Blundell, Pistaferri, and Preston (2008), Kaplan and Violante (2010) and Heathcote, Storesletten, and Violante (2014) who studied the role of advance information in standard incomplete markets environments. Blundell et al. (2008) pointed out that advance information may result in counterfactual non-zero correlations of current consumption growth with future income growth. This is problematic because these correlations serve as a key indicator for consumption smoothing. Further, Kaplan and Violante (2010) showed that the resulting increase in risk sharing with advance information is quantitatively not important enough to account for the cross-sectional dispersion of consumption in the data. With our paper, we clarify that the effect of advance information on risk sharing depends both qualitatively and quantitatively on the particular consumption-savings model employed. While we can confirm the earlier findings on advance information in the standard incomplete markets model (Blundell et al., 2008, Kaplan and Violante, 2010), a model with endogenous solvency constraints due to limited enforcement leads to very different conclusions. Without inducing counterfactual correlations of current consumption with future income growth, a limited enforcement model with advance information bridges the gap

[2] Exemplary papers in that literature are Cunha and Heckman (2016), Cunha, Heckman, and Navarro (2005), Guvenen (2007), Guvenen and Smith (2014), Huggett, Ventura, and Yaron (2006), Primiceri and van Rens (2009).

to several consumption insurance measures observed in the data.

Employing a structural-analytical framework, Heathcote et al. (2014) considered two different type of shocks, “uninsurable shocks” and “insurable shocks”. The former shocks can only be smoothed via labor supply decisions, self insurance in a non-state contingent bond or via government interventions. The latter type of shocks can (by assumption) be perfectly insured and can be interpreted as perfectly forecastable shocks through the lens of a standard incomplete markets model. Our contribution is to highlight that when households use a large variety of insurance possibilities perfectly forecastable shocks does not necessarily enhance but may actually restrict the degree of risk sharing.

Guvenen and Smith (2014) employ a life-cycle standard incomplete markets model and Bayesian indirect inference to jointly estimate how much households initially know about their future income (when learning about the nature of their income process during their working life) and the degree of partial insurance households can achieve. As main substantive finding and similar to us, the authors argue that households’ perceived income uncertainty is smaller than what is typically considered in macroeconomic models. Our work is complementary to Guvenen and Smith (2014). First, we model advance information in general equilibrium with an infinite time horizon while they consider a partial equilibrium setting with a finite time horizon. Second, we employ an explicit insurance model in which advance information can result in less insurance while in their environment advance information leads to more insurance.

Methodologically, our paper draws heavily on Alvarez and Jermann (2000), Kehoe and Levine (1993) and Krueger and Perri (2006, 2011) who studied the theoretical and quantitative properties of constrained efficient allocations with limited contract enforcement. Aiyagari (1994) pioneered in characterizing invariant distributions of consumption and assets in the standard incomplete markets model in general equilibrium. Building on these papers, Broer (2013) compared the cross-sectional implications of both models with the data. We extend the limited contract enforcement model and the standard incomplete markets model with a role for information to study how households’ perceived income uncertainty – instead of the uncertainty assessed by an econometrician – affects consumption risk sharing of U.S. households.

Hirshleifer (1971) was among the first to point out that better information makes risk-averse agents ex-ante worse off if such information leads to evaporation of risks that otherwise could have been shared in a competitive equilibrium with full insurance and perfect contract enforcement. Schlee (2001) provided conditions under which better public information about idiosyncratic risk is undesirable. Similar to these authors, we also find that better public information can result in less risk sharing. The difference is that the negative effect relies on the importance of the limited enforceability of contracts and arises only when consumption insurance is not full but partial. If enforcement frictions were absent and insurance full, information would not affect consumption allocations in the limited commitment model.

The remainder of the paper is organized as follows. In the next section, we present our economic environment. In Section 3, we provide analytical results on the effects of information on conditional and unconditional consumption moments. Section 4 is devoted to the quantitative implications of advance information for risk sharing of U.S. households. The last section concludes.

2 Environment

Preferences and endowments Consider an economy with a continuum of households indexed by i. The time is discrete and indexed by t from zero onward. Households have preferences over consumption streams and evaluate it conditional on the information available at $t = 0$

$$U\left(\{c_t^i\}_{t=0}^{\infty}\right) = (1-\beta)\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i), \tag{1}$$

where the instantaneous utility function $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, strictly concave and satisfies the Inada conditions.

Household i's disposable income in period t is given by a stochastic process $\{y_t^i\}_{t=0}^{\infty}$, where the set of possible income realizations in each period is time-invariant and finite $y_t^i \in Y \equiv \{y_1, ..., y_N\} \subseteq \mathbb{R}_{++}$, ordered. The history y^t is $(y_0, ...y_t)$. The income is independent across households and evolves across time according to a first-order Markov chain with time-invariant transition matrix $\pi_{jk} > 0$ for all j, k whose elements are the conditional probabilities of next period's income y_k given current period income y_j. The Markov chain induces a unique invariant distribution of income $\pi(y)$ such that average income (or aggregate labor endowment) is $\bar{y} = \sum_y y\pi(y)$.

Information and utility allocation Each period $t \geq 0$, household i receives a public signal $k_t^i \in Y$ that informs about income realizations in the next period. The signal has as many realizations as income states and its precision κ is captured by the probability that signal and future income coincide, $\kappa = \pi(y_{t+1} = y_j | k_t = y_j)$, $\kappa \in [1/N, 1]$. Uninformative signals are characterized by precision $\kappa = 1/N$, perfectly informative signals by $\kappa = 1$. The realizations of the public signal follow an exogenous first-order Markov process with the same conditional probabilities as income, i.e., $\pi(k' = y_j | k = y_i) = \pi(y' = y_j | y = y_i)$. Hence, at each point in time the agents can find themselves in one of the states $s_t = (y_t, k_t)$, $s_t \in S$, where S is the Cartesian product $Y \times Y$ and $s^t = (y^t, k^t) = (s_0, ...s_t)$ is the history of the state.

Using the assumptions on income and signals, the probabilities for the distribution of future income conditional on today's state s is given by[3]

$$\pi(y'|s) = \pi\left(y' = y_j | k = y_m, y = y_i\right) = \frac{\pi_{ij} \kappa^{\mathbf{1}_{m=j}} \left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=j}}}{\sum_{z=1}^{N} \pi_{iz} \kappa^{\mathbf{1}_{m=z}} \left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=z}}} \quad , \tag{2}$$

where $\mathbf{1}_{m=j}$ is an indicator function and equals one if the signal and the actual realization of income coincide. The logic of the formula is a signal extraction with two independent signals on future income realizations, current income and the public signal. Both signals enter the signal extraction weighted with their precision, income with transition probability π_{ij} and signals with precision κ.

For example, with uninformative signals ($\kappa = 1/N$) the conditional expectation of income y_j tomorrow given signal k_j and income y_i can be computed as

$$\pi\left(y' = y_j | k = y_j, y = y_i\right) = \frac{\pi_{ij} \frac{1}{N}}{\frac{1}{N} \sum_{z=1}^{N} \pi_{iz}} = \pi_{ij}.$$

With signals following an exogenous process, the conditional distribution of signals and income can be combined to a time-invariant Markov transition matrix P_s with conditional probabilities $\pi(s'|s)$ as elements

$$\pi(s'|s) = \pi\left(y' = y_j, k' = y_l | k = y_m, y = y_i\right) = \pi(k' = y_l | k = y_m) \pi\left(y' = y_j | k = y_m, y = y_i\right). \tag{3}$$

[3] Appendix A.6 provides details on the derivation of the formulas for the joint distribution of income and signals. Further, we provide arguments for the assumptions on the signal process.

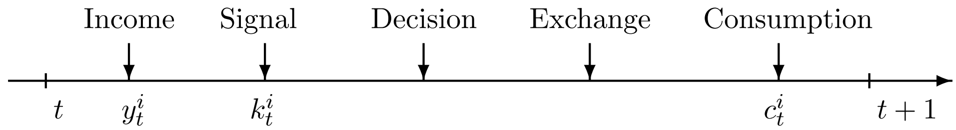

Figure 1: Timing of events with limited contract enforcement and public information

Households differ with respect to their initial utility entitlements w_0 and the initial state s_0 which constitute their information set in period $t = 0$. The utility allocation is $h = \{h_t(w_0, s^t)\}_{t=0}^{\infty}$ and the consumption allocation c can be obtained as $c = \{C\left(h_t(w_0, s^t)\right)\}_{t=0}^{\infty}$, where $C : \mathbb{R} \to \mathbb{R}_+$ is the inverse of the period utility function u. Thus, the allocation depends on initial utility promises and on the history of income and signals s^t.

Insurance arrangements To protect their consumption from undesirable fluctuations, households can trade insurance contracts that cover the complete state space but have limited enforceability (or limited commitment) as in Kehoe and Levine (1993) and Krueger and Perri (2011). As illustrated in Figure 1, enforcement of the insurance contracts is limited because each period after receiving the current income and the public signal, households decide to participate in risk-sharing contracts implementing the allocation c or to deviate into autarky forever, consuming only their endowment.[4] The limited enforceability of contracts gives rise to participation constraints for each history s^t for all periods t:

$$(1-\beta)u\left(C(h_t)\right) + \beta \sum_{s_{t+1}} \pi(s_{t+1}|s^t)\, \mathrm{U}_{t+1}\left(\{C(h_\tau)\}_{\tau=t+1}^{\infty}\right) \geq U^{Aut}(s_t), \tag{4}$$

with $h_t = h_t(w_0, s^t)$, $h_\tau = h_\tau(w_0, s^\tau)$ and

$$U^{Aut}\left(s_t\right) \equiv (1-\beta)u(y_t) + \beta \sum_{y_{t+1}} \pi(y_{t+1}|s_t)\, \mathrm{U}_{t+1}\left(\{y_\tau\}_{\tau=t+1}^{\infty}\right) \tag{5}$$

as the value of the outside option (autarky) and $U_\tau = (1-\beta)\, \mathrm{E}_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} u\left(c_t\right)$.

Efficient allocations Let $\Phi_0 = \Phi_0(w, s)$ be an initial distribution over w_0 and the initial shocks $s_0 = (y_0, k_0)$. In the following definitions, we capture constrained feasible and efficient allocations.

[4] In quantitative analysis, we allow also for self insurance in the outside option.

Definition 1 *An allocation* $\{h_t(w_0, s^t)\}_{t=0}^{\infty}$ *is constrained feasible if*

(i) the allocation delivers the promised utility w_0

$$w_0 = \mathrm{U}\left(\{C(h_t)\}_{t=0}^{\infty}\right);$$

(ii) the allocation satisfies for each state s^t *participation constraints* (4)

(iii) and the allocation is resource feasible

$$\sum_{s^t} \int \left[C(h_t) - y_t\right] \pi(s^t|s_0) d\Phi_0 \leq 0 \quad \forall t. \tag{6}$$

Atkeson and Lucas (1992) show by applying a duality argument that efficient allocations can be computed by minimizing resource costs to deliver the promises made in Φ_0. The notion of efficiency is summarized in the following definition.

Definition 2 *An allocation* $\{h_t(w_0, s^t)\}_{t=0}^{\infty}$ *is efficient if it is constrained feasible and there does not exist another constrained feasible allocation* $\{\hat{h}_t(w_0, s^t)\}_{t=0}^{\infty}$ *with respect to* Φ_0 *that requires fewer resources in one period* t

$$\exists t: \quad \sum_{s^t} \int \left[C(\hat{h}_t) - C(h_t)\right] \pi(s^t|s_0) \, \mathrm{d}\, \Phi_0 < 0.$$

Summing up, an efficient allocation respects promises and participation incentives, is resource feasible and there is no other allocation that uses fewer resources that is also constrained feasible.

Stationary efficient allocations As in Krueger and Perri (2011), we restrict ourselves to computing stationary efficient allocations – allocations in which the distribution of current utility and utility promises is constant across time. As shown by Atkeson and Lucas (1992), a stationary allocation is efficient if it is a solution to a dynamic programming problem (described in the next paragraph) and if it satisfies resource feasibility. In the next section, we will show that these allocations can be also interpreted as allocations stemming from a competitive equilibrium.

Given utility promise w, state $s = (y, k)$ and an inter-temporal price $R \in (1, 1/\beta]$, a planner (or financial intermediary) chooses a portfolio of current utility h and future promises $w'(s')$ for each future income realization y' and signal k'. The portfolio $(h, \{w'(s')\})$ is required to minimize the discounted resources costs, to deliver the promised value $w(s)$ and to satisfy participation constraints. Formally, the minimization problem reads as follows

$$\begin{aligned} V(w,s) &= \min_{h,\{w'(s')\}} \left[\left(1-\frac{1}{R}\right) C(h) + \frac{1}{R} \sum_{s'} \pi(s'|s) V\left(w'(s'), s'\right) \right] \quad (7) \\ & \quad \text{s.t.} \\ w &= (1-\beta)h + \beta \sum_{s'} \pi(s'|s) w'(s') \quad (8) \\ w'\left(s'\right) &\geq U^{Aut}\left(s'\right), \forall s'. \quad (9) \end{aligned}$$

The solution is characterized by the first order conditions:

$$V_w\left(w'(s'), s'\right) \frac{1-\beta}{(R-1)\beta} \geq C'(h) \quad (10)$$

$$V_w\left(w'(s'), s'\right) \frac{1-\beta}{(R-1)\beta} = C'(h), \quad \textit{if } w'(s') > U^{Aut}(s'), \quad (11)$$

where we have applied the envelope condition

$$\lambda = \frac{\partial V(w,s)}{\partial w} = \frac{R-1}{(1-\beta)R} C'(h), \quad (12)$$

where λ is the Lagrange multiplier associated with the promise keeping constraint (8). The result of the minimization problem are policy functions $h(w,s), \{w'(w,s;s')\}$. Following similar arguments as in Krueger and Perri (2011), $w' \in W = [\underline{w}, \overline{w}]$, $\underline{w} = \min_{s'} U^{Aut}(s')$ and $\overline{w} = \max_{s'} U^{Aut}(s')$. The state space therefore comprises $Z = W \times S$ with Borel σ algebra $\mathcal{B}(Z)$ and typical subset $(\mathcal{W}, \mathcal{S})$. The Markov process for income and signals P_s together with the policy functions w' generate a law of motion for the probability measure $\Phi_{w,s}$. Let $Q_R\left[(w,s),(\mathcal{W},\mathcal{S})\right]$ be the probability that an agent with utility promise w and income-signal state s transits to the set $\mathcal{W} \times \mathcal{S}$:

$$Q_R\left[(w,s),(\mathcal{W},\mathcal{S})\right] = \sum_{s' \in \mathcal{S}} \begin{cases} \pi(s'|s) & \text{if } w'(w,s) \in \mathcal{W} \\ 0 & \text{else} \end{cases}$$

It follows that the law of motion for the probability measure $\Phi_{w,s}$ is given by

$$\Phi'_{w,s} = H(\Phi_{w,s}) = \int Q_R\left[(w,s),(\mathcal{W},\mathcal{S})\right]\Phi_{w,s}(\mathrm{d}\,w \times \mathrm{d}\,s).$$

An allocation is stationary if $\Phi_{w,s} = H(\Phi_{w,s}) = \Phi_0$.[5]

Definition 3 *An efficient allocation in the limited commitment economy is a stationary utility allocation $\{h(s,w)\}$ and an invariant probability measure $\Phi_{w,s}$ induced by the problem (7)-(9) satisfying resource feasibility* (6)*.*

3 Analysis

In this section, we provide analytical results on the effect of public information on consumption risk sharing. First, we describe how efficient allocations with public information can be decentralized in a competitive equilibrium. As the key result in this section, we show that better public information reduces risk-sharing possibilities in the limited commitment model and thus results in a riskier consumption distribution when risk sharing is partial. Further, we analyze how the precision of public signals affect the conditions for full perfect risk sharing and autarky as efficient allocations.

3.1 Competitive equilibrium and decentralization

In a decentralized version of the economy, households are heterogenous in initial asset holdings, income and signals. We start with defining a competitive equilibrium as in Krueger and Perri (2011) that follows Kehoe and Levine (1993) and derive prices to decentralize the efficient allocations. Denote by $p_t(s^t)$ the period-zero price of a unit of period-t consumption faced by a household following history s^t. A household with initial wealth a_0, initial endowment y_0, and signal k_0 chooses an allocation $\{c_t(a_0,s^t)\}_{t=0}^{\infty}$ that provides the highest utility subject to their intertemporal budget constraint

$$c_0(a_0,s_0) + \sum_{t=1}^{\infty}\sum_{s^t|s_0} p_t(s^t)c_t(a_0,s^t) \leq y_0 + \sum_{t=1}^{\infty}\sum_{s^t|s_0} p_t(s^t)y_t + a_0 \tag{13}$$

[5] The probability measure can be shown to exist and to be unique. The proof relies on an argument that requires the state space of promises to be compact which follows from analogous arguments as in Krueger and Perri (2011).

and their participation incentives for each history s^t in each period t

$$(1-\beta)u\left(c_t\right)+\beta\sum_{s_{t+1}}\pi(s_{t+1}|s^t)\,\mathrm{U}\left(\{c_\tau\}_{\tau=t+1}^{\infty}\right)\geq$$
$$(1-\beta)u(y_t)+\beta\sum_{y_{t+1}}\pi(y_{t+1}|s^t)\,\mathrm{U}_{t+1}\left(\{y_\tau\}_{\tau=t+1}^{\infty}\right), \quad (14)$$

where $c_t = c_t(a_0, s^t)$ and $c_\tau = c_\tau(a_0, s^\tau)$.

Definition 4 *A competitive equilibrium with limited commitment is a price system $\{p_t(s^t)\}_{t=0}^{\infty}$ and an allocation $\{c_t(a_0, s^t)\}_{t=0}^{\infty}$ such that*

(i) given prices, the allocation of each household (a_0, s_0) solves the household's problem;

(ii) all markets clear.

An efficient allocation with public information can be decentralized as a competitive equilibrium which is captured in the following proposition.

Proposition 1 *A stationary efficient allocation $\{C(h_t(w_0, s^t))\}_{t=0}^{\infty}$ can be decentralized as a competitive equilibrium allocation $\{c_t(a_0, s^t)\}_{t=0}^{\infty}$ with prices and initial asset holdings given by*

$$p_t(s^t)=\frac{\pi(s^t|s_0)}{R^t} \quad \textit{and} \quad a_0=c(w_0,s_0)-y_0+\sum_{t=1}^{\infty}\sum_{s^t|s_0}\frac{\pi(s^t|s_0)}{R^t}\left[c(w_0,s^t)-y_t\right].$$

The proof is provided in Appendix A.1. This proposition is important for two reasons. First, we can interpret the stationary allocations stemming from the recursive social planner problem (7)-(9) more realistically as allocations generated by a competitive equilibrium. Second, the distribution of assets across households instead of utility promises offers the possibility to analyze the effects of public signals on wealth inequality.

3.2 Information, perfect risk sharing and autarky

To analytically characterize the effects of public information, we abstract in this section from a number of features. We assume that the set of possible income realizations consists of two states only $y_h = \bar{y} + \delta_y$ and $y_l = \bar{y} - \delta_y$, where $\delta_y > 0$ is the standard deviation of the income process. Further, the states are equally likely and the income realizations are independent across time and agents. Correspondingly, public signals are i.i.d. as well and can indicate either a high income ("good" or "high" signals) or a low income ("bad" or "low" signals) in the future. Efficient allocations may feature either full insurance, partial insurance or no insurance against income risk (autarky). The first case is analyzed in the following proposition.

Proposition 2 (Perfect Risk Sharing) *Consider efficient allocations.*

1. *There exists a unique cutoff point,* $0 < \bar{\beta}(\kappa) < 1$, *such that for any discount factor* $1 < \beta \geq \bar{\beta}(\kappa)$ *the efficient allocation for any signal precision is perfect risk sharing.*

2. *The cutoff point* $\bar{\beta}(\kappa)$ *is increasing in the precision of the public information.*

The proof is provided in Appendix A.2. The cutoff point for perfect risk sharing is determined by the tightest participation constraint which is the one of the high income agent receiving a high signal on future income. The long-term gains from risk sharing can only outweigh the desire to leave the arrangement if agents are sufficiently patient. Furthermore, the value of the outside option at the tightest participation constraint is increasing in the precision of the signal.

On the other extreme, autarky may be the only constrained feasible allocation and therefore efficient. In the next proposition, we provide conditions for this case.

Proposition 3 (Autarky) *Autarky is efficient if and only if*

$$\beta < \frac{u'(y_h)}{u'(y_l)}.$$

The proof is provided in Appendix A.3. Thus, with public information the condition for autarky to be efficient is identical to the condition in absence of public information analyzed in Krueger and Perri (2011). The empirically relevant case is the one when risk sharing is neither perfect nor absent but partial which we analyze below.

3.3 Information and partial risk sharing

In this sub-section, we analytically characterize how better public information affects the unconditional and conditional distribution of consumption with partial risk sharing. As the key result here, we find that better public information has a negative effect on risk sharing.

We continue to employ on an i.i.d. income with two equally likely states. Further, we consider allocations that do not depend on the history of the state s^t but only on the current state s_t. We refer to such allocations as memoryless allocations which are defined as follows.

Definition 5 *An allocation* $\{h_t(w_0, s^t)\}_{t=0}^{\infty}$ *is a memoryless allocation (denoted* h_{ML}*) if:*

$$\forall s^t \quad \{h_t(w_0, s^t)\}_{t=0}^{\infty} = \{h_t(s_t)\}_{t=0}^{\infty} \equiv h_{ML}.$$

While these allocation are in general not efficient, they allow us to prove the effects of public information on the consumption distribution analytically. A utility allocation in this simplified setting is $h_{ML} = \left\{u\left(c_i^j\right)\right\}$ where for notational convenience we have $c_i^j = C(h(y = y_j, k = y_i))$. Social welfare is

$$V_{rs} = (1-\beta)\frac{1}{4}\sum_{t=0}^{\infty}\sum_{j\in\{l,h\}}\sum_{i\in\{l,h\}}\beta^t u\left(c_{i,t}^j\right). \tag{15}$$

Resource feasibility requires in each period t

$$\frac{1}{4}\sum_{j\in\{l,h\}}\sum_{i\in\{l,h\}} c_{i,t}^j = \frac{1}{2}\sum_{j\in\{l,h\}} y_{j,t}. \tag{16}$$

Participation is constrained by rational incentives that take into account the public signal. For illustrative purposes, we focus here only on high-income agents. The participation constraints are thus for a good public signal

$$\begin{aligned}(1-\beta)u(c_h^h) + \beta(1-\beta)\left[\kappa V_{rs}^h + (1-\kappa)V_{rs}^l\right] + \beta^2 V_{rs} \geq V_{h,out}^h \\ = (1-\beta)u(y_h) + \beta(1-\beta)\left[\kappa u(y_h) + (1-\kappa)u(y_l)\right] + \beta^2 V_{out}, \end{aligned}\tag{17}$$

and for a bad public signal

$$\begin{aligned}(1-\beta)u(c_l^h) + \beta(1-\beta)\left[(1-\kappa)V_{rs}^h + \kappa V_{rs}^l\right] + \beta^2 V_{rs} \geq V_{l,out}^h \\ = (1-\beta)u(y_h) + \beta(1-\beta)\left[(1-\kappa)u(y_h) + \kappa u(y_l)\right] + \beta^2 V_{out}, \end{aligned}\tag{18}$$

with

$$V_{rs}^h = \frac{1}{2}\left[u(c_h^h) + u(c_l^h)\right], \quad V_{rs}^l = \frac{1}{2}\left[u(c_h^l) + u(c_l^l]\right)$$

and

$$V_{rs} = \frac{1}{2}\left[V_{rs}^h + V_{rs}^l\right], \quad V_{out} = \frac{1}{2}\left(u(y_h) + u(y_l)\right).$$

As a next step, we provide the definition of an optimal allocation in this environment.

Definition 6 *An optimal memoryless allocation is a consumption allocation* $\{c_i^j\}$ *that maximizes households' utility (15) subject to resource feasibility (16) and enforcement constraints.*

In this environment, we can analytically show how unconditional and conditional moments of consumption are affected by information precision. As summarized in the following proposition, better public signals lead to less risk sharing and higher consumption dispersion.

Proposition 4 (Information and risk sharing) *Consider an optimal memoryless allocation with partial risk sharing such that enforcement constraints (17)-(18) are binding. An increase in information precision has the following effects on the consumption allocation:*

1. *The conditional mean of consumption of high-income agents increases and the conditional mean of low-income agents decreases.*

2. *The conditional standard deviation of consumption of high-income agents increases.*

3. *The unconditional standard deviation of consumption increases.*

The proof is provided in Appendix A.4.

Better public information results in lower average consumption of low-income agents and in higher average consumption but as well in higher consumption dispersion of high-income agents. This makes the consumption distribution riskier from an ex-ante perspective.

To get intuition, consider an increase in the precision of public signals. By (17) and (18), this results in an increase in the value of the outside option for high-income agents with a good public signal and a decrease for agents with a bad public signal. As captured by the changes in the outside option values, agents with a bad signal are more willing while the agents with a good signal are less willing to share their current high income. Thus, consumption of high-income agents spreads out and the conditional standard deviation of high-income increases. Thereby, the changes in the value of the outside option of high-income agents with a good signal ($V^h_{h,out}$) and with a bad signal ($V^h_{l,out}$) are symmetric:

$$\frac{\partial V^h_{h,out}}{\partial \kappa} = -\frac{\partial V^h_{l,out}}{\partial \kappa}.$$

For informative signals, the high-income agents with a good public signal have a lower marginal utility of consumption and thus require more additional resources than the high-income agents with a bad public signal are willing to give up. In sum, conditional mean consumption of high-income agents increases which by resource feasibility reduces the risk-sharing possibilities for low-income agents. As a consequence, the allocation becomes riskier ex ante and the unconditional standard deviation of consumption increases as well.

In this section, we have shown that better public information results in a riskier allocation ex ante such that the standard deviation of consumption increases. Further, better public information results in higher consumption of high-income and lower consumption of low-income agents. Thus, better public information has the potential to improve the predictions of the limited commitment model for the unconditional and conditional distribution of consumption. In the next section, we quantitatively explore whether a limited commitment model extended with a role of advance information can indeed reproduce the consumption moments observed in the data and can be used to **quantify** households unobserved perceived income risk.

4 Quantitative Results

In this section, we provide quantitative results on the effect of advance information on risk sharing of households in the United States. We start by explaining the data employed in the quantitative exercise and the calibration. Then, we first illustrate the quantitative implications of advance information for constrained-efficient consumption allocation in the endowment economy as described in Section 2. To distinguish households' perceived income uncertainty from the income uncertainty as measured by an econometrician and to compute our main quantitative results, we employ a production economy with capital and endogenous solvency constraints. As a robustness exercise, we also study the role of advance information in a standard incomplete markets model – both in an endowment and a production economy with capital.

4.1 Data and calibration

To facilitate comparison with related studies in particular to Krueger and Perri (2006) and Broer (2013), we employ the Consumer Expenditure Interview Survey (CEX), and follow these authors in their methodology. In particular, we decompose consumption and income inequality in between and within group inequality. Between-group inequality are differences in household income and consumption attributable to observable characteristics for example education, region of residence, etc., and assume that households cannot insure against these observable characteristics. Income inequality devoid of between group inequality component is called within group inequality. This residual measure of inequality is the focus of this paper as it is caused by the idiosyncratic income shocks and hence, depending on the insurance available against these shocks, consumption inequality will not exactly mirror income inequality.

As measure of household consumption, we employ non-durable consumption (ND+) which also includes an estimate for service flows from housing and cars. For households' disposable income, we use after-tax labor earnings plus transfers (LEA+). Consistent with voluntary participation, we thus take the mandatory public insurance as given and focus on private insurance. LEA+ comprises the sum of wages and salaries of all household members, plus a fixed fraction of self-employment farm and non-farm income, minus reported federal, state, and local taxes (net of refunds) and social security contributions plus government transfers.

We drop the households who report zero or only food consumption, whose head is older than 64 years or younger than 21 years, with negative or zero earnings or have negative working hours, which have positive labour income but no working hours, which live in the rural area or their weekly wage is below the minimum wage and which are not present in all interviews. To facilitate a comparison between households of different size, the consumption and income measures are divided by adult equivalence scales as in Dalaker and Naifeh (1997).

To compute within group inequality, we follow Krueger and Perri (2006) and Blundell et al. (2008), and regress the logs of household consumption and income on the set of observed characteristics dummies. The dummies include region, marital status, race, education, experience, occupation, sex and age. The residuals of the regression are treated as consumption and income shock.

Model parameters Our annual calibration is designed to highlight the differences between a standard limited commitment model without information as entertained in Broer (2013) and a model with information. Therefore, we set a number of corresponding parameters to the same values. In particular, we consider a period utility function that exhibits constant relative risk aversion with parameter $\sigma = 1$. The discount factor β is chosen to yield an annual gross interest rate of $R = 1.025$ in general equilibrium. In the production economy with capital in Section 4.3, we further use a Cobb-Douglas production function $AF(K, L)$ with a capital-production elasticity of 0.30. Given R, we choose the depreciation of the capital stock δ and the technology parameter A to yield a real wage rate of unity and an aggregate wealth-to-income ratio of 2.5 as for example estimated by Kaplan and Violante (2010) based on the Survey of Consumer Finances (SCF).

Following the practice in the literature, the income specification comprises persistent and transitory income components. Log income of household i is modeled as

$$\ln(y_{it}) = z_{it} + \epsilon_{it}, \qquad z_{it} = \rho z_{it-1} + \eta_{it},$$

where ϵ_{it} and η_{it} are independent, serially uncorrelated and normally distributed with variances σ_ϵ^2 and σ_η^2, respectively. The persistence parameter ρ is set to 0.9989 which is the value originally found by Storesletten et al. (2004). Given the persistence parameter, we identify the variances $\sigma_\epsilon^2, \sigma_\eta^2$ from the cross-sectional within-group income variance and auto-covariance in the CEX data as the averages of the years 1999–2003. The method proposed by Tauchen and Hussey (1991) is used to approximate the persistent part of income by a Markov process with three states and time-invariant transition probabilities, and the transitory part is modeled with two exogenous states of equal probability. We normalize the value of all income states such that mean income (or aggregate labor endowment) is equal to 1. For each of the 6 income states, there are therefore 6 public signals such that the joint income-signals state S is approximated by 36 states which is higher than the 14 states typically considered in related studies (Broer, 2013, Krueger and Perri, 2006). The increase in the number of states leads to a numerical challenge for computing consumption allocations in general equilibrium.[6]

Insurance measures To measure the extent of consumption smoothing from the data, we focus on two measures: (1) the covariance of consumption and income growth, and (2) relative variance of log-consumption with respect to log-income. The first measure captures the sensitivity of consumption growth to income growth. Following Mace (1991), the sensitivity is captured by the coefficient $\beta_{\Delta y}$ in the following regression equation

$$\Delta c_{it} = \psi + \beta_{\Delta y} \Delta y_{it} + \upsilon_t + \nu_{it} \tag{19}$$

where ψ is a constant, υ_t a vector of time dummies and ν_{it} a residual; Δc_{it} and Δy_{it} are the growth rates of consumption and income of individual i in period t. When the coefficient $\beta_{\Delta y}$ is zero, then consumption growth is perfectly insured against changes in income growth. The higher is the coefficient, the less insurance is achieved.

The second measure is defined as one minus the ratio of the cross-sectional unconditional variance of logged consumption over logged income:

$$RS = 1 - \frac{\text{var}_c}{\text{var}_y} \tag{20}$$

On one extreme, if $\text{var}_c = \text{var}_y$, then $RS = 0$, and there is no private insurance against fluctuations in disposable. On the other hand, if $\text{var}_c = 0$ then $RS = 1$ implying full insurance against income shocks. In Table 1, we summarize the calibrated parameters in the upper part and unconditional moments of consumption and income from the CEX data in the lower part. The value of $\beta_{\Delta y}$ is equal to 11 percent with a standard error of 0.0035; the insurance ratio is $1 - \frac{\text{var}_c}{\text{var}_y} = 0.60$ which implies 40 percent of income shocks transfer to consumption.

Information and reduction in perceived income uncertainty To interpret the effects of an increase in information precision κ, we compute the percentage reduction of households' perceived income uncertainty $\tilde{\kappa}$ as measured by the reduction in the mean-squared forecast error resulting

[6] In Appendix A.7, we describe our algorithm for computing allocations in the *LC* model in more detail. With 500 points on the promises grid, we solve in each iteration step for 666,000 variables. In a standard model without information and 14 income states as in Broer (2013), the corresponding number of variables is 105,000.

Table 1: Baseline parameters and CEX moments

	Parameter	Value
σ	Risk aversion	1
α	Elasticity of capital in production function	0.3000
R	Gross interest rate	1.0250
ρ	Auto-correlation	0.9989
var_z	Variance persistent	0.2505
var_ϵ	Variance transitory	0.1149
S	Income-signal states	36
var_y	Variance log income	0.3654
var_c	Variance log consumption	0.1462
$\beta_{\Delta y}$	Regression coefficient	0.1078

from conditioning expectations on signals

$$\tilde{\kappa} = \frac{\text{MSFE}_y - \text{MSFE}_s}{\text{MSFE}_y}, \tag{21}$$

with

$$\text{MSFE}_y = \sum_y \pi(y) \sum_{y'} \pi(y'|y) \left[y' - \text{E}(y'|y)\right]^2$$

$$\text{MSFE}_s = \sum_s \pi(s) \sum_{y'} \pi(y'|s) \left[y' - \text{E}(y'|s)\right]^2,$$

and $\pi(s)$ as the joint invariant distribution of income and signals. Thus, $\tilde{\kappa}$ captures the difference in income uncertainty as measured by an econometrician in the aggregate and the income uncertainty as perceived by households. For this reason, we refer to $\tilde{\kappa}$ as the uncertainty gap. If signals are uninformative, $\tilde{\kappa}$ is equal to zero and if signals are perfectly informative, $\tilde{\kappa}$ equals one. For a given increase in κ, the reduction in perceived income risk is the smaller the more persistent income is.

Outside option For the quantitative results, we allow for self insurance in the outside option. In case of defaulting to the outside option and consistent with U.S. bankruptcy law, households loose all their consumption claims. Further, access to financial markets is restricted. While agents can save unlimited amounts in a non-state contingent bond with gross return $R^{Aut} > 0$, they cannot borrow. Thus, the value of the outside option is a solution to an optimal savings problem that can be written in recursive form as follows

$$v(s,a) = \max_{0 \le a' \le y + aR^{Aut}} \left[(1-\beta) u(aR^{Aut} + y - a') + \beta \sum_{s'} \pi(s'|s) v'(s', a') \right].$$

We set $R^{Aut} = R$, implying that in the outside option households can save at the intertemporal price $1/R$ also used in the recursive problem (7)-(9). Thus, the value of the outside option will be conditional on R and is given by

$$U_R^{Aut}(s) = v(s,0).$$

4.2 Advance information and constrained-efficient allocations

In the simplified environment of the analytical section, we showed that the unconditional variance of consumption is increasing in information precision. Thus, insurance of income shocks is decreasing in information precision because the consumption allocation becomes riskier *ex-ante*. We find that this negative effect of information on insurance also applies in the fully-fledged environment with efficient history-dependent insurance contracts and a persistent income process. Further, the negative effect of information on risk sharing is quantitatively important for the unconditional and income-conditional moments of consumption.

In Figure 2, we plot on the left axis the unconditional variance of logged consumption and on the right axis the resulting insurance ratio in the stationary equilibrium. As shown in the third part of Proposition 4, the unconditional variance of consumption is increasing in the precision of information, or equivalently, is increasing in the difference between the income uncertainty as measured by an econometrician and the income uncertainty as perceived by households. Correspondingly, the insurance ratio on the right axis decreases when income risk is reduced. In the standard model without advance information ($\tilde{\kappa} = 0$), the insurance ratio is equal to 83 percent which is 23 percent higher than the 60 percent observed in the CEX data. This finding is typical for the limited commitment model that is known for predicting too much consumption insurance for standard calibrations. The insurance ratio is sensitive with respect to advance information. Reducing the income risk by 18 percent, results in an insurance ratio of 0.58.

Figure 3 illustrates the effect of advance information on the conditional mean and standard deviation of consumption for low-and high-income households. The results confirm to a large

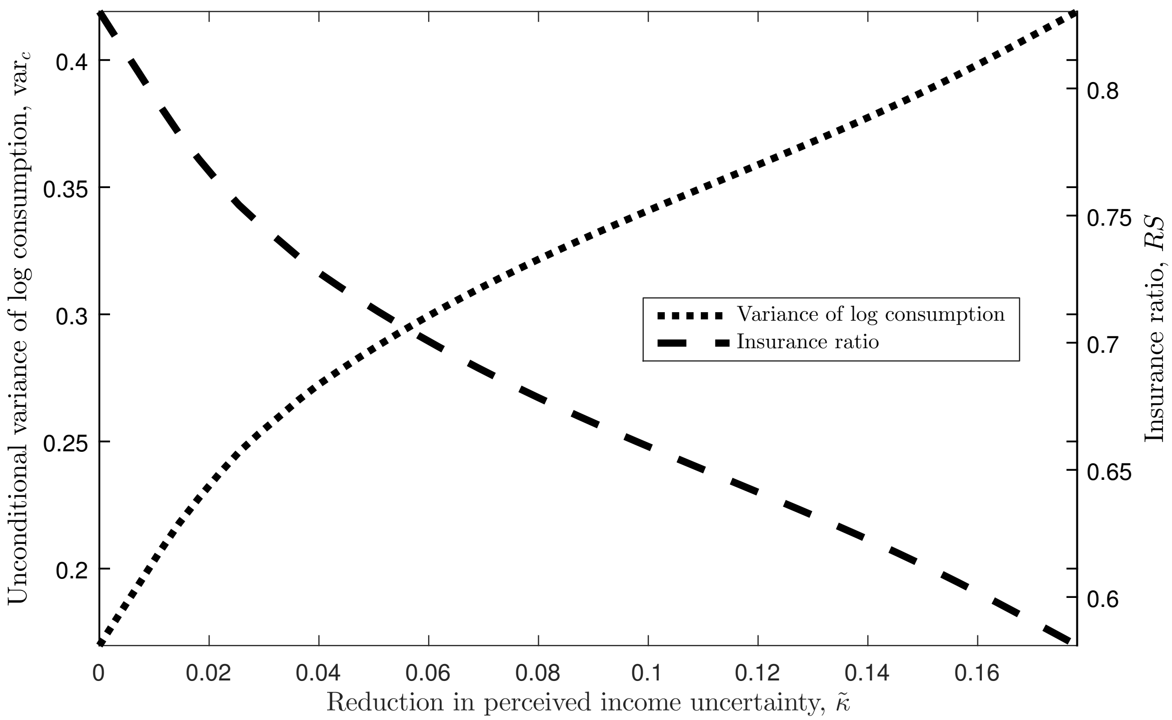

Figure 2: Limited commitment endowment economy. The $x-$axis picks the reduction in perceived income uncertainty, $\tilde{\kappa}$; the left $y-$axis represents the unconditional variance of consumption; the right $y-$axis the insurance ratio.

extent the analytical results of the first and second part of Proposition 4 derived in the simplified environment of the previous section. However, there is one important exception. With memoryless allocations, the conditional standard deviation of low-income households is equal to zero, in efficient allocations the standard deviation is positive.

While the conditional mean consumption of low-income households decreases, consumption of high-income households increases when income risk is resolved by advance information. When we decentralize efficient allocations as described in Section 3.1, the changes in consumption are reflected in corresponding changes in assets holdings across income groups; mean asset holdings of low-income households decrease while high-income households hold more assets on average when income risk is resolved (see the lower panel of Figure 3). Further, the conditional standard deviation of consumption increases for low-and high-income households. Quantitatively, the standard deviation of low-income households is affected stronger than the dispersion of consumption among high-income households: it increases by 25 log points while for high-income agents, the increases equals 6 log points when households' income risk reduces by 18 percent.

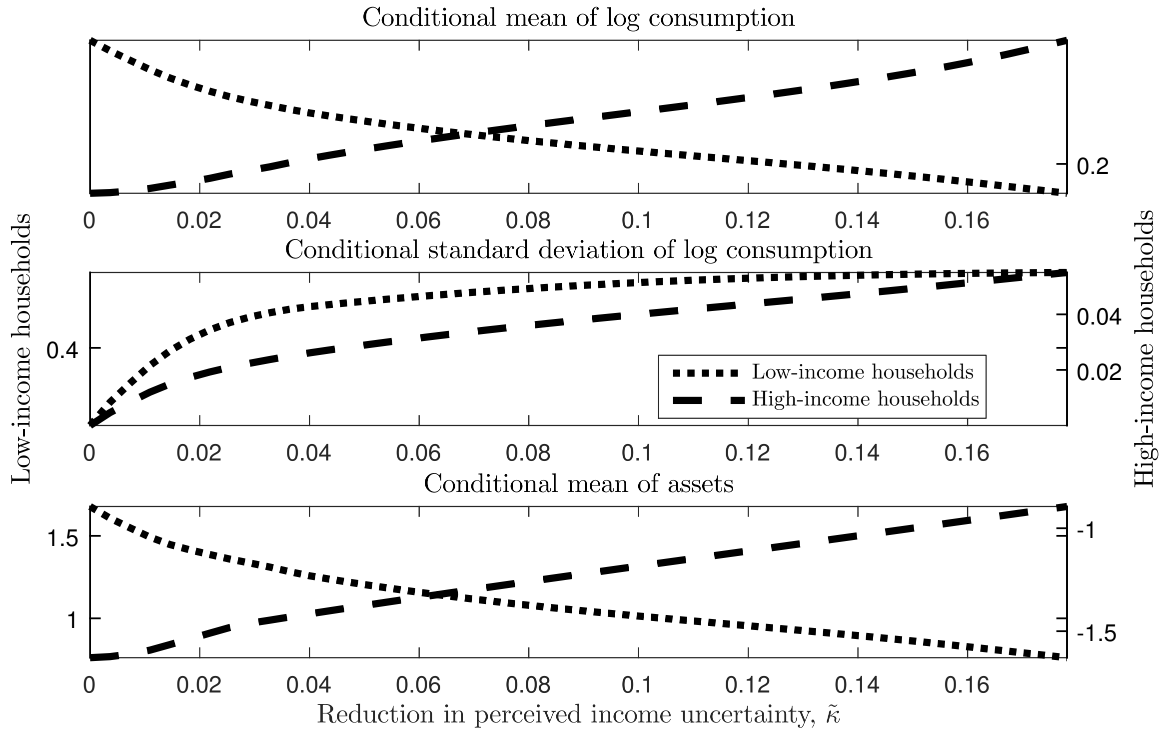

Figure 3: Limited commitment endowment economy. The $x-$axis picks the reduction in perceived income uncertainty, $\tilde{\kappa}$; the left $y-$axis represents low-income households, the right $y-$axis high-income households. The upper panel is the conditional mean of log consumption, the middle panel is the conditional standard deviation of log consumption and the lower panel pictures the conditional mean of assets holdings.

To facilitate comparison with existing studies without advance information, we now continue with a production economy with capital and limited commitment.

4.3 A stationary production economy with endogenous solvency constraints

The production economy with endogenous solvency constraints is in detail explained in Krueger and Perri (2006). In the following, we refer to this economy as the *limited commitment production economy*. Here, we directly focus on stationary allocations. Households trade a complete set of one-period zero coupon assets $a(s')$ priced at $q(s, s')$ with financial intermediaries that live for one period and invest into capital. As in Alvarez and Jermann (2000), households face state-contingent endogenous credit limits $A(s')$ that are not "too tight". Given asset holdings a, state $s = (y, k)$, and prices $w, \{q(s,s')\}$, households' problem can be written recursively as

$$V(a,s) \quad = \quad \max_{c,\{a'(s')\}} \left[(1-\beta)u(c) + \beta \sum_{s'} \pi(s'|s) V(a'(s'), s')\right]$$

subject to a budget and solvency constraints

$$c + \sum_{s'} q(s,s')a'(s') \quad \leq \quad wy + a \tag{22}$$

$$a'(s') \quad \geq \quad A(s'), \quad \forall s', \tag{23}$$

where wy is labor income. The endogenous credit limits are pinned down by outside option values $U^{aut}(s')$

$$A(s') = \min_{a(s')} \left\{a(s') : V'(a(s'), s') \geq U^{Aut}(s')\right\}, \quad \forall s'. \tag{24}$$

The result of the maximization problem are policy functions $c(a,s), \{a'(a,s;s')\}$.

Here, households differ with respect to initial asset asset holdings and initial shocks where the heterogeneity is captured by the invariant probability measure $\Phi_{a,s}$.

A representative firm hires labor L and capital K at rental rates w and r to maximize profits where the production of consumption goods Y takes place via a linear homogenous production function

$$Y = AF(L,K),$$

where A is a constant productivity parameter, L is the aggregate labor endowment (in efficiency units) and K the aggregate capital stock. Capital depreciates at rate δ.

The stationary recursive competitive equilibrium is summarized in the following definition.

Definition 7 *A stationary recursive competitive equilibrium in the limited commitment production economy comprises a value function $V(a,s)$, a price system $R, w, q(s,s')$, an allocation $K, c(a,s), \{a'(a,s;s')\}$, a joint probability measure of assets and state $\Phi_{a,s}$, and endogenous credit limits $A(s')$ such that*

(i) $V(a,s)$ is attained by the decision rules $c(a,s)$, $\{a'(a,s;s')\}$ given $R, w, q(s,s')$

(ii) Endogenous credit limits are determined by outside option values according to (24)

(iii) The joint distribution of assets and state $\Phi_{a,s}$ induced by $\{a'(a, s; s)\}$ and P_s is stationary

(iv) No arbitrage applies

$$q(s, s') = \frac{\pi(s'|s)}{R}$$

(v) Factor prices satisfy

$$R - 1 = AF_K(1, K) - \delta$$

$$w = AF_L(1, K)$$

(vi) The asset market clears

$$R'K' = \int \sum_{s'} a'(a, s; s')\pi(s'|s) \, \mathrm{d}\, \Phi_{a,s}.$$

Quantifying advance information To discipline the free parameter κ, we choose the parameter such that the risk sharing predicted by the model matches two distinct insurance measures observed in the data. The insurance ratio as the first measure characterizes the cross-sectional dispersion of consumption. As the second measure, we employ the regression coefficient of current consumption growth with respect to income growth as a measure to determine the sensitivity of consumption with respect to changes in income. In general, we therefore expect to pin down two values for the reduction in households' perceived income uncertainty $\tilde{\kappa}_1, \tilde{\kappa}_2$ that yield insurance measures in the model that are consistent with the measure observed in the CEX.

For the first insurance measure, we use the cross-sectional variance of consumption in the invariant distribution. For the second insurance measure, we employ stationarity and simulate the model for 300,000 time periods and discard the first 100,000 periods to ensure convergence. Then we estimate correlations of consumption and income growth using the simulated data.

As displayed in Figure 4, households' advance information has a strong quantitative effect on consumption insurance that allows us to capture both insurance measures with advance information. Without information, risk sharing is nearly perfect such that the insurance coefficients equals one (left axis) and the regression coefficients β_{Δ_y} is close to zero (right axis).[7] The insurance ratio is

[7] Krueger and Perri (2006) provide similar results for the standard model without information. With capital,

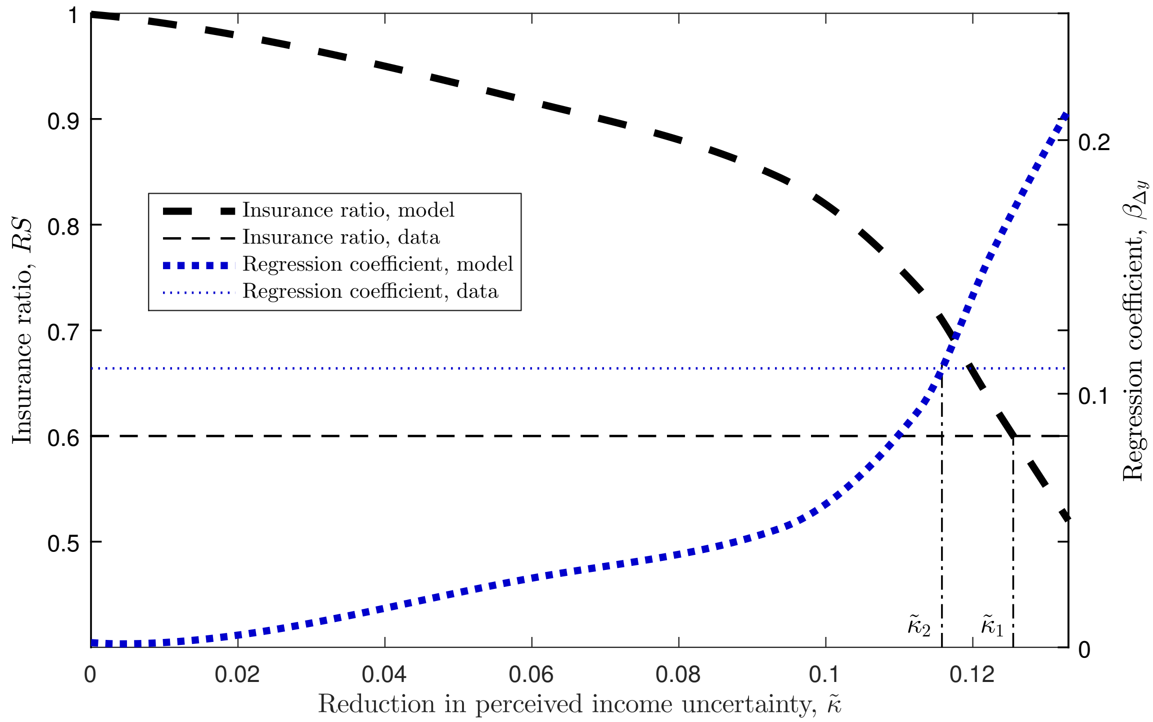

Figure 4: Limited commitment production economy. The $x-$axis picks the reduction in perceived income uncertainty $\tilde{\kappa}$; the left $y-$axis represents the insurance ratio; the right $y-$axis the regression coefficient of current consumption growth on current income growth; the black dashed line is the insurance ratio of 0.60, the black doted line is the regression coefficient of 0.11 from the CEX, 1998–2003.

matched for a reduction in income uncertainty of $\tilde{\kappa}_1 = 0.1236$, the regression coefficient is matched for $\tilde{\kappa}_2 = 0.1158$. Thus, both insurance measures are jointly matched for a reduction in income risk of 12 percent. This is a remarkable result because in general the two insurance measures have to coincide only in the extreme cases when risk sharing is either perfect or absent.

Blundell et al. (2008) argue that including advance information in the SIM model may lead to correlations of current consumption with future income growth that are not consistent with the data. To test for a potential role of advance knowledge of future income shocks, Blundell et al. (2008) used household panel data from the Panel Study of Income Dynamics (PSID) to estimate correlations of current consumption growth $\Delta c_{i,t} = \log(c_t^i) - \log(c_{t-1}^i)$ with future income growth $\Delta y_{i,t+j} = \log(y_{t+j}^i) - \log(y_{t+j-1}^i)$ for $j \geq 1$. Through the lens of a standard incomplete markets

risk sharing improves compared to an endowment economy because in autarky households loose their claims on the capital stock which decreases the value of their outside option. To receive partial insurance with capital, Broer (2013) introduces the possibility that households can with a probability of 12 percent return to insurance after defaulting. Using this return probability, we compute $RS = 0.89$ and $\beta_{\Delta y} = 0.06$.

model, if there was advance knowledge of income shocks, the correlation in the data should be significantly different from zero because consumption should adjust before the shock has occurred. However, Blundell et al. (2008) estimate correlations that are not significantly different from zero with p-values larger than 0.25.

The limited commitment model with information is consistent with that evidence. As reported in the first column of Table 2, the correlation of current consumption growth with future income growth is not significantly different from zero for the standard model with $\tilde{\kappa} = 0$. This pattern does not change for informative signals. As displayed in the second and third column, for $\tilde{\kappa}_1 = 0.1236$ (yields the insurance ratio from the data) and for $\tilde{\kappa}_2 = 0.1158$ (yields the regression coefficient from the data), only the correlation of current income growth and current consumption growth is significantly different from zero. Consistent with Blundell et al. (2008), the correlations of current consumption growth with future income growth are not significantly different from zero with p-values larger than 89 percent. Unlike in a standard incomplete markets model, advance information in the LC model does not induce counterfactual correlations of current consumption growth with future income growth.[8]

The logic for this result can be intuitively rationalized in the limited commitment endowment economy. In the limited commitment model, the size of the income uncertainty is not directly relevant because in principle there is a complete set of securities available for insurance. Consumption insurance is imperfect because the enforcement of insurance contracts is limited by the outside option to live in autarky. In the optimal insurance contract with partial insurance, the planner encourages high-income agents with binding enforcement constraints to transfer resources today in exchange for insurance of income shocks in the future. Insurance involves both promising to decouple future income and future consumption (insurance across states), and to smooth consumption across periods. This logic is strengthened further by more precise signals. When signals become more precise, the outside option becomes more attractive for agents with a high income. To accommodate this change and to encourage these agents to transfer resources today, the planner promises even more consumption smoothing across time and states. Nevertheless, according to Proposition 4, transfers from high-to-low income agents are reduced such that the dependency of

[8] The CEX is a revolving panel in which households drop out after one year. For this reason, we cannot estimate correlations of current consumption with future income growth.

Table 2: Income and consumption growth regression: limited commitment production economy

	No signals, $\tilde{\kappa} = 0.00$	$\tilde{\kappa}_2 = 0.1158$	$\tilde{\kappa}_1 = 0.1236$	**CEX Data**
$\beta_{\Delta y_t}$	0.00	0.11	0.16	**0.11**
β_2	−0.00	0.01	0.01	–
Test $\text{cov}(\Delta c_t, \Delta y_t), p$-values	0.00	0.05	0.05	**0.00**
Test $\text{cov}(\Delta c_t, \Delta y_{t+1}), p$-values	0.00	0.89	0.92	–

Notes: In the table, we provide regression coefficients and their p values for the regression $\Delta c_t^i = \beta_0 + \beta' \Delta y^i + \epsilon_t^i$, with $\beta = [\beta_{\Delta y_t}, \beta_2]'$ and $\Delta y^i = [\Delta y_t^i, \Delta y_{t+1}^i]'$ for different precisions of signals.

current consumption and current income strengthens while more consumption smoothing across periods prevents a higher correlation of current consumption with future income growth.

Aguiar and Bils (2015) and Attanasio et al. (2012) argued that the consumption expenditures reported in the CEX Interview Survey are exposed to measurement error that result in biased estimates of cross-sectional consumption inequality measures. In particular, Aguiar and Bils (2015) claimed that the insurance ratio might be smaller than the 60 percent as directly measured using the CEX data that we employ as one of the insurance measures. Nevertheless, the uncertainty gap we identify is not very sensitive with respect to a potentially downward-biased estimate for consumption inequality for two reasons. First, using the regression coefficient as an alternative insurance measure yields very similar numbers for advanced information. This measure is less prone to measurement error because the regression coefficient employs growth rates as a ratio and therefore corrects for time-invariant multiplicative measurement error. Additionally, we find that the regression coefficient in the CEX of 0.11 is very close to the corresponding coefficient that we estimate using PSID data of 0.12. Second, even if the correct insurance ratio was higher than the number computed directly from the CEX, the identified uncertainty gap would only be mildly affected. As illustrated in Figure 4, consumption insurance reacts sensitively to advance information. For example, if the insurance ratio was with 50 percent lower than the 60 percent as directly observed in the CEX, the identified reduction in perceived income uncertainty changes only mildly from 12 to 13 percent. In that sense, the uncertainty gap of 12 percent can also be interpreted as a lower bound.

Kaplan and Violante (2010) conclude that advance information cannot reconcile insurance ratios or regression coefficients in a life-cycle standard incomplete markets model and in the data. In Section 4.5, we also allow for signals in a standard infinite horizon incomplete markets (SIM)

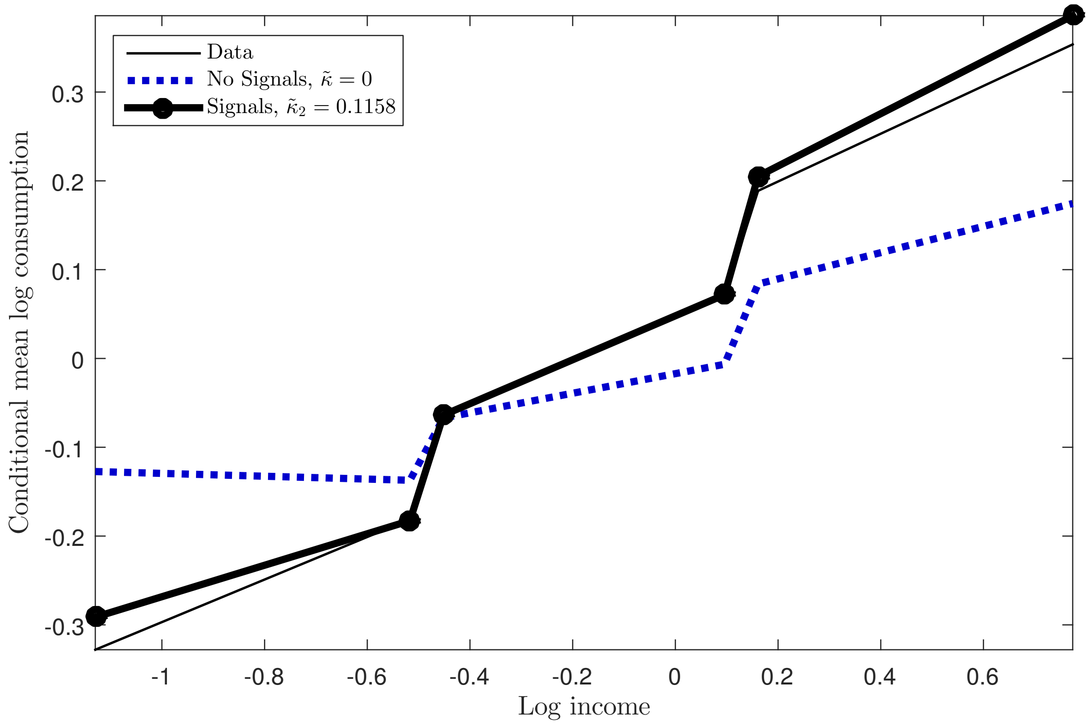

Figure 5: Limited commitment production economy. Conditional mean of logged consumption with respect to logged income for different precisions of signals. The $x-$axis represents the log income and $y-$axis represents the conditional mean of log consumption. Income steps represent percentiles: $[17th, 33th, 50th, 67th, 83th]$. Solid line captures the conditional means for the years 1999–2003 in the CEX.

model and confirm the earlier findings of Kaplan and Violante (2010).

We find that the picture changes when we alternatively employ the limited commitment model with explicit insurance contracts. Here, advance information on future income shocks can bridge the gap to the data because risk sharing in the limited commitment model is more sensitive to information than in the SIM model. Further, advance information does not induce counterfactual correlations of current consumption growth with future income growth. Correspondingly, we can quantify households' advance information by matching the insurance ratio or, alternatively, by capturing the regression coefficient of consumption on income growth. Our main quantitative finding is that both insurance measure can be jointly explained when households' perceived income uncertainty is reduced by 12 percent. In the following, we fix information precision at this value, and analyze the implications for the joint distribution of income and consumption as "over-identifying restrictions".

4.4 Information and conditional moments of consumption

To compare conditional moments from the data and models, our procedure is the following. We start with the stationary distribution of income implied by the Tauchen and Hussey (1991)'s procedure and compute the conditional mean and variance corresponding to this stationary distribution in each model. For the data, we employ the percentiles from the stationary income distribution and compute the moments for the percentiles, accordingly. For our calibration, this corresponds to the following percentiles: $[17th, 33th, 50th, 67th, 83th]$. For example, households with a high income represent the top 17 percent of income earners.

Throughout this section, we use the conditional consumption moments as over-identifying restrictions and compare the standard model without signals to the case of informative signals. For informative signals, we consider precisions $\tilde{\kappa}_1 = 0.1236$ and $\tilde{\kappa}_2 = 0.1158$ that capture the insurance ratio and the regression coefficient β_{Δ_y}. Insurance is however perfect in the standard limited commitment production model without signals. To facilitate a fair comparison, we employ the results derived in the endowment economy for the standard model.[9]

In Figure 5, we plot the conditional mean of log consumption for the data, standard model and for informative signals of precision $\tilde{\kappa}_2 = 0.1158$. In the absence of signals, the average consumption of low-income households is too high compared to the data while the consumption of high-income agents is too low. Further, indicating also too much insurance for low-income states, average consumption is constant for the two low-income groups in the absence of information; in the CEX data, average consumption is increasing for all income states. With informative signals, household consumption becomes more dispersed. Consistent with the first part of Proposition 4, we find that average consumption of low-income households decreases while consumption of high-income households increases, leading to a more dispersed consumption distribution and a better fit to the data. Further and as in the data, the conditional mean of consumption is increasing in income over all incomes states. Overall, the conditional mean of consumption is tracked in an almost perfect way for informative signals over all six income groups.

As displayed in Figure 6, advance information results in a higher conditional standard deviation for all income groups. In particular, information leads to an increase in consumption dispersion

[9] Alternatively employing the standard model with the possibility to return from autarky to insurance as in Broer (2013) yields similar conditional consumption moments.

Table 3: Conditional moments of consumption: model versus data

	No signals, $\tilde{\kappa} = 0$	$\tilde{\kappa}_2 = 0.1158$	$\tilde{\kappa}_1 = 0.1236$	**CEX Data**
MSE, $\mathrm{E}\left[\log(c)\vert y\right]$, normalized	34.15	1	4.67	–
$\mathrm{E}\left[\log(c)\vert y_h\right] - \mathrm{E}\left[\log(c)\vert y_l\right]$	0.30	0.68	0.80	**0.68**
MSE, $\mathrm{STD}\left[\log(c)\vert y\right]$, normalized	3.48	1	0.91	–
$\frac{\mathrm{STD}[\log(c)\vert y_h]}{\mathrm{STD}[\log(c)\vert y_l]}$	0	0.38	0.36	**0.95**

Notes: The table provides the mean squared deviations of model and data for the conditional means and standard deviations of consumption expressed relative to signals with $\tilde{\kappa}_2 = 0.1158$; the table also provides spreads between average consumption and the standard deviation of low-and high income households.

conditional on a high income; in the absence of information, the standard deviation is zero while with informative signals it is positive and increasing in precision. While the conditional standard deviation is tracked reasonably well for low-and middle-income earners, the distance to the data increases of higher income groups.

Quantitatively, the fit of the conditional consumption distribution to the data is substantially improved by advance information. As displayed in Table 3, the mean-squared deviations of the conditional mean of consumption between model and data are approximately 34 times as large in the standard model than for $\tilde{\kappa}_2 = 0.1158$; for $\tilde{\kappa}_1 = 0.1236$, the mean deviations are 4.5 times higher than for $\tilde{\kappa}_2 = 0.1158$ but still over 7 times lower than in the standard model. Further, the spread between average consumption of high-and low income households in the CEX data of 0.68 is perfectly captured by signals with $\tilde{\kappa}_2 = 0.1158$.

There is also some improvement in fit for the conditional standard deviation of consumption but the improvement is not as striking as for the conditional mean. Relative to the standard model, the mean-square error is 3.5 times smaller for $\tilde{\kappa}_2 = 0.1158$, and approximately 4 times smaller for $\tilde{\kappa}_1 = 0.1236$. Further, the ratio of the conditional standard deviations for high-and low-income households increases from 0 in the standard model to 0.4 with advance information. This increase is however too small to capture the ratio of almost 1 observed in the CEX.

4.5 Advance information in a standard incomplete markets model

As a robustness exercise, we integrate public signals in a standard incomplete markets (SIM) model. Confirming earlier results by Kaplan and Violante (2010), we find little support for a prominent role of advance information in the SIM-model.

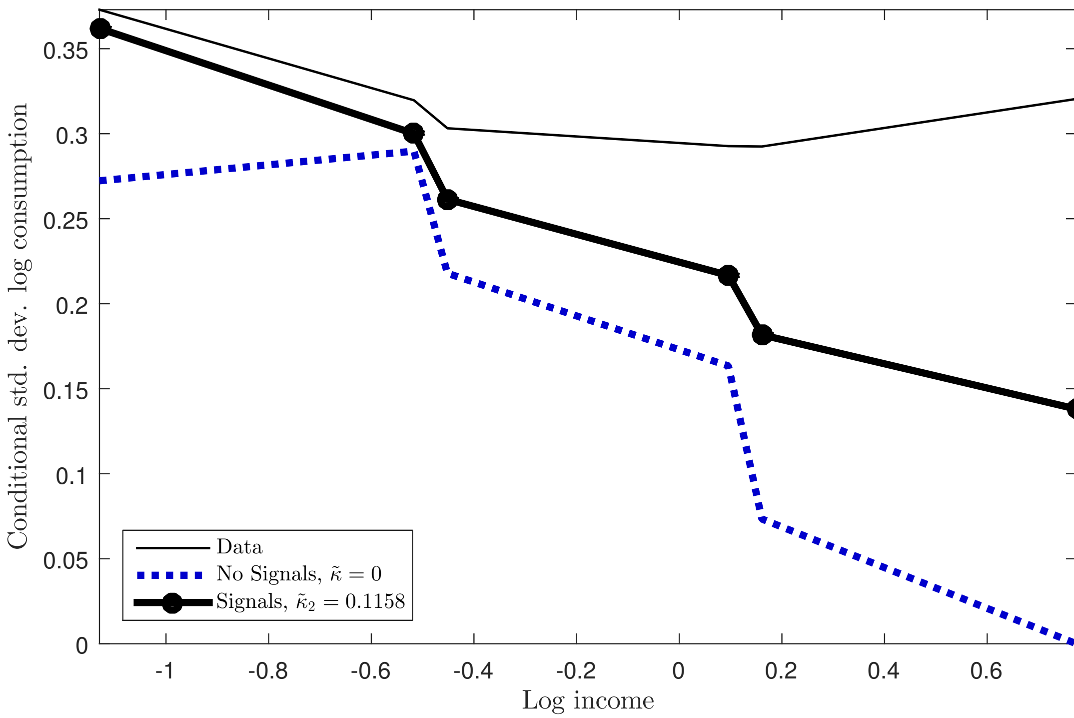

Figure 6: Limited commitment production economy. Conditional standard deviation of logged consumption with respect to logged income for different precisions of the signals. The $x-$axis represents the log income and the $y-$axis the conditional standard deviation of logged consumption. Income steps represent percentiles: $[17th, 33th, 50th, 67th, 83th]$. Solid line captures the conditional standard deviations for the years 1999–2003 in the CEX.

Environment While preferences and endowments are as described in Section 2, households in the standard incomplete markets economy can only trade in a single non-state contingent bond with gross return R and face an exogenous borrowing limit $\bar{a}$. There are no enforcement frictions and we directly focus on stationary allocations. The model we consider is similar to Huggett (1993) and relies on a market structure with a continuum of households as in Aiyagari (1994). Given asset holdings a, state $s = (y, k)$, and an interest rate R, households' problem can be written recursively as

$$V(a, s) \quad = \quad \max_{c, a'} \left[(1 - \beta) u(c) + \beta \sum_{s'} \pi(s'|s) V(a', s') \right]$$

subject to a budget and a borrowing constraint

$$\begin{aligned} c + a' &\leq y + Ra \\ a' &\geq -\bar{a}. \end{aligned}$$

Here, households differ with respect to initial asset asset holdings and initial shocks where the heterogeneity is captured by the probability measure $\Psi_{a,s}$. The state space is given by $M = A \times S$, where $A = [-\bar{a}, \infty)$.

The stationary recursive competitive equilibrium is summarized in the following definition.

Definition 8 *A stationary recursive competitive equilibrium in the standard incomplete markets economy comprises a value function $V(a,s)$, an inter-temporal price R, an allocation $c(a,s), a'(a,s)$ a joint probability measure of assets and the state $\Psi_{a,s}$, and an exogenous borrowing limit $\bar{a}$ such that*

(i) $V(a,s)$ is attained by the decision rules $c(a,s)$, $a'(a,s)$ given R

(ii) The joint distribution of assets and state $\Psi_{a,s}$ induced by $a'(a,s)$ and P_s is stationary.

(iii) The bond market clears

$$\int a'(a,s)\,\mathrm{d}\,\Psi_{a,s} = 0.$$

Quantitative results As emphasized by Blundell et al. (2008) and Kaplan and Violante (2010), in a SIM model better information on future income realizations allows households to improve on their consumption-savings decisions, and risk sharing improves. Thus, better information here has a positive effect by improving individual decision which is referred to as a Blackwell (1953) effect of information. For generating the quantitative results, we employ for for the common parameters the same parameter values as in the corresponding limited commitment economy. Wolff (2011) finds that 19 percent of all U.S. households are borrowing constrained. For this reason, we choose an exogenous borrowing limit $\bar{a}$ to yield in equilibrium 19 percent borrowing-constrained households in the standard model without information.

In line with earlier findings by Kaplan and Violante (2010), we find that insurance ratios improve monotonically in information precision but the improvement is too small to capture the insurance ratio of 60 percent observed in the data even for very informative signals. In the absence of signals, the model implies that households insure about 25 percent of all fluctuations in their after-tax income. As an extreme case, if information precision amounts to $\kappa = 0.99$ – corresponding to a reduction of income uncertainty $\tilde{\kappa}$ of 97 percent, the insurance ratio reaches 0.33. Thus, the increase in insurance by better information is quantitatively too small to capture the insurance observed in CEX data.

Further as displayed in Table 4, as signals become informative the SIM model predicts that current consumption growth is counter-factually correlated with future income growth. For uninformative signals and informative signals with precisions up to $\tilde{\kappa} = 0.75$, current consumption growth is uncorrelated with income growth one period ahead on a 10-percent significance level (see the first three columns). However, the regression coefficient of current consumption with current income growth of 0.17 is still too high compared to the 0.11 estimated in the CEX data. From $\tilde{\kappa} = 0.76$ onwards, the correlation of current consumption growth with income growth one period in the future is statistically significantly different from zero and with a coefficient of $\beta_2 = 0.12$ also economically significant (see the forth column). The non-zero correlation is inconsistent with the evidence provided in Blundell et al. (2008) who find correlations of current consumption growth with future income growth not significantly different from zero. Further even for $\tilde{\kappa} = 0.97$, the regression coefficients is with 0.15 too high compared to the data.

The logic behind the non-zero correlation of current consumption with future income growth in the SIM model can be rationalized as follows. In the SIM model, better information reduces directly the income fluctuations households want to insure. Knowing future income allows for better insurance of income risk given the limited option to use a non-state contingent bond. Thus, before the shock realizes households' consumption today reacts to the part of the future income shock that is known, and consumption today is correlated with future income when signals become precise enough.

For $\tilde{\kappa} = 0.75$ as the highest value for $\tilde{\kappa}$ that yields no counterfactual correlation of current consumption with future income growth, the insurance ratio of 32 percent however falls short compared to the 60 percent as observed in the CEX data.

Table 4: Income and consumption growth regression: SIM model

	No signals, $\tilde{\kappa} = 0.00$	$\tilde{\kappa} = 0.16$	$\tilde{\kappa} = 0.76$	**CEX Data**
$\beta_{\Delta y_t}$	0.28	0.25	0.17	**0.11**
β_2	−0.02	0.00	0.12	–
Test $\mathrm{cov}(\Delta c_t, \Delta y_t), p$-values	0.00	0.00	0.02	**0.00**
Test $\mathrm{cov}(\Delta c_t, \Delta y_{t+1}), p$-values	0.77	0.99	0.10	–

Notes: In the table, we provide regression coefficients and their p values for the regression $\Delta c_t^i = \beta_0 + \beta' \Delta y^i + \epsilon_t^i$, with $\beta = [\beta_{\Delta y_t}, \beta_2]'$ and $\Delta y^i = [\Delta y_t^i, \Delta y_{t+1}^i]'$.

A production economy with capital Alternatively, we consider capital as a physical asset instead of the financial asset a in the endowment economy. Instead of (iii) in Definition 8, asset market clearing now requires

$$\int a'(a, s)\, \mathrm{d}\, \Psi_{a,s} = K'.$$

Further, the rental prices of the production factors equal their marginal products. For all values of $\tilde{\kappa}$, the capital-to-income ratio is kept constant at 2.5 and the wage is normalized to unity. As before, the exogenous borrowing limit is chosen to yield 19 percent borrowing-constrained households with uninformative signals.

We compute quantitative similar results on the effect of advance information as in the endowment economy. The insurance ratio RS increases from 37 percent without information to 44 percent with signals that reveal 97 percent of all future income uncertainty. Thus, the increase in insurance amounts to 7 percent (compared to 8 percent in the endowment economy). The regression coefficient $\beta_{\Delta y}$ decreases from 0.26 without advance information to 0.15 for signals with $\tilde{\kappa} = 0.97$. As in the endowment economy however, from $\tilde{\kappa} = 0.76$ onwards, current consumption growth is correlated with future income growth with an economically and statistically significant coefficient of 0.11.

5 Conclusions

In this paper, we have developed a framework to address the issue of a potential disconnect between households' income uncertainty and the income uncertainty as measured by an econometrician raised by Browning, Hansen, and Heckman (1999) and Cunha and Heckman (2016). To that end, we have developed a risk sharing model that can distinguish between the two types of uncertainties in a systematic and consistent way. To quantify the difference in the perception of uncertainty, we have employed a general equilibrium model with endogenous borrowing constraints. Using U.S. micro data, we have found that there is a systematic uncertainty gap: households' perceived income uncertainty is 12 percent lower than the uncertainty estimated by an econometrician that is typically used in consumption risk sharing models. For this uncertainty gap, the model jointly explains three distinct consumption insurance measures that are not captured in the absence of advance information: (i) the cross-sectional variance of consumption, (ii) the covariance of consumption with income growth, and (iii) the income-conditional mean of household consumption.

With their recent paper, Heathcote, Storesletten, and Violante (2016) contribute to a lively debate on the optimal progressivity of taxes in the United States. One of the main arguments in favor for a progressive tax system is that it helps to insure idiosyncratic earnings uncertainty when private insurance is limited. Thereby, a higher tax progressivity reduces the earnings risk after taxes. Computing the optimal tax progressivity requires a precise estimate for households' earnings uncertainty. In particular, if there is a systematic uncertainty gap as suggested in this paper and income uncertainty is actually lower than what is typically considered, less tax progressivity might be desirable than conventional wisdom suggests.

One of the limitations of this paper is that the precision of households' advance information is not a choice variable. One possible avenue for future research is to consider the precision of signals a choice variable; higher precision of signals could for example result in higher resource, utility, or time costs. Then the precision of signals can differ across households in the population reflecting the financial situation of a household and the importance of better information on future earnings for making accurate consumption-savings decisions.

References

Aguiar, M. and M. Bils (2015): "Has Consumption Inequality Mirrored Income Inequality?" *American Economic Review*, 105, 2725–56.

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109, 659–684.

Alvarez, F. and U. J. Jermann (2000): "Efficiency, Equilibrium, and Asset Pricing with Risk of Default," *Econometrica*, 68, 775–798.

Atkeson, A. and R. E. Lucas (1992): "On Efficient Distribution with Private Information," *Review of Economic Studies*, 59, 427–453.

Attanasio, O., E. Hurst, and L. Pistaferri (2012): "The Evolution of Income, Consumption, and Leisure Inequality in the US, 1980-2010," NBER Working Papers 17982, National Bureau of Economic Research, Inc.

Blackwell, D. (1953): "Equivalent comparisons of experiments," *Ann. Math. Stat.*, 24, 265–272.

Blundell, R., L. Pistaferri, and I. Preston (2008): "Consumption Inequality and Partial Insurance," *American Economic Review*, 98, 1887–1921.

Broer, T. (2013): "The wrong shape of insurance? What cross-sectional distributions tell us about models of consumption smoothing," *American Economic Journal: Macroeconomics*, 5, 107–140.

Browning, M., L. P. Hansen, and J. J. Heckman (1999): "Micro data and general equilibrium models," in *Handbook of Macroeconomics*, ed. by J. B. Taylor and M. Woodford, Elsevier, vol. 1 of *Handbook of Macroeconomics*, chap. 8, 543–633.

Cunha, F. and J. Heckman (2016): "Decomposing Trends in Inequality in Earnings into Forecastable and Uncertain Components," *Journal of Labor Economics*, 34, S31–S65.

Cunha, F., J. Heckman, and S. Navarro (2005): "Separating uncertainty from heterogeneity in life cycle earnings," *Oxford Economic Papers*, 57, 191–261.

Dalaker, J. and M. Naifeh (1997): "Poverty in the United States," *US Bureau of the Census, Current Population Reports. Series P60-201. Washington, DC: US Government Printing Office.*

Guvenen, F. (2007): "Learning Your Earning: Are Labor Income Shocks Really Very Persistent?" *American Economic Review*, 97, 687–712.

Guvenen, F. and A. A. Smith (2014): "Inferring Labor Income Risk and Partial Insurance From Economic Choices," *Econometrica*, 82, 2085–2129.

Heathcote, J., K. Storesletten, and G. L. Violante (2014): “Consumption and Labor Supply with Partial Insurance: An Analytical Framework,” *American Economic Review*, 104, 1–52.

——— (2016): “Optimal Tax Progressivity: An Analytical Framework,” *Quarterly Journal of Economics*, Forthcoming.

Hirshleifer, J. (1971): “The Private and Social Value of Information and the Reward to Inventive Activity,” *American Economic Review*, 61, 561–574.

Huggett, M. (1993): “The risk-free rate in heterogeneous-agent incomplete-insurance economies,” *Journal of Economic Dynamics and Control*, 17, 953–969.

Huggett, M., G. Ventura, and A. Yaron (2006): “Human capital and earnings distribution dynamics,” *Journal of Monetary Economics*, 53, 265–290.

Kaplan, G. and G. L. Violante (2010): “How Much Consumption Insurance beyond Self-Insurance?” *American Economic Journal: Macroeconomics*, 2, 53–87.

Kehoe, T. J. and D. K. Levine (1993): “Debt-Constrained Asset Markets,” *The Review of Economic Studies*, 60, 865–888.

Krueger, D. and F. Perri (2006): “Does Income Inequality Lead to Consumption Inequality? Evidence and Theory,” *Review of Economic Studies*, 73, 163–193.

——— (2011): “Public versus Private Risk Sharing,” *Journal of Economic Theory*, 146, 920–956.

Lepetyuk, V. and C. A. Stoltenberg (2013): “Policy Announcements and Welfare,” *Economic Journal*, 123, 962–997.

Mace, B. J. (1991): “Full Insurance in the Presence of Aggregate Uncertainty,” *Journal of Political Economy*, 99, 928–956.

Primiceri, G. E. and T. van Rens (2009): “Heterogeneous Life-Cycle Profiles, Income Risk and Consumption Inequality,” *Journal of Monetary Economics*, 56, 20–39.

Schlee, E. E. (2001): “The Value of Information in Efficient Risk-Sharing Arrangements,” *American Economic Review*, 91, 509–524.

Storesletten, K., C. I. Telmer, and A. Yaron (2004): “Cyclical Dynamics in Idiosyncratic Labor Market Risk,” *Journal of Political Economy*, 112, 695–717.

Tauchen, G. and R. Hussey (1991): “Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models,” *Econometrica*, 59, 371–96.

Wolff, E. N. (2011): "Recent Trends in Household Wealth, 1983-2009: The Irresistible Rise of Household Debt," *Review of Economics and Institutions*, 2.

A Appendix

A.1 Proof of Proposition 1

The first order condition for competitive equilibrium consumption $c_t = c_t(a_0, s^t)$ requires

$$\beta^t \pi(s^t|s_0)u'(c_t) \leq \lambda p_t(s^t),$$

where λ is the Lagrange multiplier on the intertemporal budget constraint (13). Asset prices are determined by households with the highest willingness to pay for the asset. These are unconstrained households. For two consecutive periods, the first order conditions for households with slack participation constraints are

$$\beta^t \pi(s^t|s_0)u'(c_t) = \lambda p_t(s^t),$$
$$\beta^{t+1} \pi(s^{t+1}|s_0)u'(c_{t+1}) = \lambda p_{t+1}(s^{t+1}).$$

Dividing those we obtain:

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} = \frac{\pi(s^t|s_0)p_{t+1}(s^{t+1})}{p_t(s^t)\pi(s^{t+1}|s_0)} \tag{25}$$

Consider a consumption allocation from the planner problem $\{C(h_t(w_0, s^t))\}_{t=0}^{\infty}$. The allocation is an equilibrium allocation if it satisfies the optimality condition of a household with non-binding participation constraints in periods t and $t+1$ over history s^{t+1}:

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} = \frac{1}{R}. \tag{26}$$

Combining the optimality conditions of planner and households results in

$$\frac{1}{R} = \frac{\pi(s^t|s_0)p_{t+1}(s^{t+1})}{p_t(s^t)\pi(s^{t+1}|s_0)},$$

which implies that

$$p_t(s^t) = \frac{\pi(s^t|s_0)}{R^t}.$$

Finally, the initial wealth that makes the allocation (w_0, s_0) affordable is given by

$$a_0 = c(w_0, s_0) - y_0 + \sum_{t=1}^{\infty} \sum_{s^t|s_0} \frac{\pi(s^t|s_0)}{R^t} \left[c(w_0, s^t) - y_t\right].$$

A.2 Proof of Proposition 2

Let $\bar{V}_{rs} = u(\bar{y})$ be the period-social welfare under perfect risk sharing. First, perfect risk sharing provides the highest *ex-ante* utility among the consumption-feasible allocations. The existence of $\bar{\beta}(\kappa)$ follows from monotonicity of participation constraints in β and $\bar{V}_{rs} > V_{out}$. A higher β increases the future value of perfect risk sharing relative to the allocation in the equilibrium without transfers, leaving the current incentives to deviate unaffected. Therefore, if the participation constraints are not binding for $\bar{\beta}(\kappa)$, they are not binding for any $\beta \geq \bar{\beta}(\kappa)$. The cutoff point is characterized by the tightest participation constraint, i.e., by the participation constraint with the highest value of the outside option. Solving this constraint yields a unique solution for $\bar{\beta}(\kappa)$ in $(0, 1)$ because $u(\bar{y}) < u(y_h)$. Second, the tightest constraint at the first best allocation is

$$u(\bar{y}) \geq (1-\beta)u(y_h) + \beta(1-\beta)\kappa u(y_h) + \beta(1-\beta)(1-\kappa)u(y_l) + \beta^2 V_{out}.$$

Differentiating the constraint fulfilled with equality with respect to κ using the implicit function theorem gives

$$\frac{\partial \bar{\beta}(\kappa)}{\partial \kappa} = \frac{\beta(1-\beta)[u(y_h) - u(y_l)]}{(1-\beta)[u(y_h) - \kappa u(y_h) - (1-\kappa)u(y_l)] + \beta[\kappa u(y_h) + (1-\kappa)u(y_l) - u(y_l)]} \geq 0$$

and results in a positive sign for $y_h > y_l$.

A.3 Proof of Proposition 3

First, if $\beta \leq u'(y_h)/u'(y_l)$, autarky is a solution to the recursive problem because (10) is satisfied for autarky. From the other end, we have to show that it is not possible to construct a resource-feasible distribution that dominates the stationary distribution $\Phi^{Aut}(\{U^{Aut}(y), y\}) = q(y)$ when the condition in the proposition is satisfied. Consider the following distribution $\hat{\Phi}$ over utility promises, income and public signals with a one-period history

$$\hat{\Phi}(\{U^{Aut}(y_h,k_h),y_h\})=\pi_h/2,\quad \hat{\Phi}(\{U^{Aut}(y_l,k_h),y_l\})=(1-\pi_h)(1-\pi_h)/2,$$
$$\hat{\Phi}(\{U^{Aut}(y_l,k_l),y_l\})=(1-\pi_h)(1-\pi_h)/2\quad \hat{\Phi}(\{U^{Aut}(y_h,k_l),y_h\})=\pi_h/2$$
$$\hat{\Phi}(\{\tilde{\omega}^{k_h}_{k_h},y_l\})=(1-\pi_h)\pi_h/4\quad \hat{\Phi}(\{\tilde{\omega}^{k_h}_{k_l},y_l\})=(1-\pi_h)\pi_h/4$$
$$\hat{\Phi}(\{\tilde{\omega}^{k_l}_{k_h},y_l\})=(1-\pi_h)\pi_h/4\quad \hat{\Phi}(\{\tilde{\omega}^{k_l}_{k_l},y_l\})=(1-\pi_h)\pi_h/4$$

where $\tilde{\omega}^{k_j}_{k_i}=U^{Aut}(y_l,k_i,n_h)+\varepsilon^{k_j}_{k_i}$ for small $\varepsilon^{k_j}_{k_i}$ and and the upper (lower) index indicates the previous (current) period signal.

Let $\{\delta^m_{ij}\}$, $i,j,m\in\{l,h\}$ denote transfers in terms of utility where the first index is current income, the second index denotes the current public signal and the upper index is previous period public signal. The transfers are defined by

$$\tilde{\omega}^{k_h}_{k_h}=(1-\beta)(u(y_l)+\delta^h_{lh})+\beta\left[(1-\kappa)U^{Aut}(y_l)+\kappa U^{Aut}(y_h)\right]$$
$$\tilde{\omega}^{k_l}_{k_h}=(1-\beta)(u(y_l)+\delta^l_{lh})+\beta\left[(1-\kappa)U^{Aut}(y_l)+\kappa U^{Aut}(y_h)\right]$$
$$\tilde{\omega}^{k_h}_{k_l}=(1-\beta)(u(y_l)+\delta^h_{ll})+\beta\left[\kappa U^{Aut}(y_l)+(1-\kappa)U^{Aut}(y_h)\right]$$
$$\tilde{\omega}^{k_l}_{k_l}=(1-\beta)(u(y_l)+\delta^l_{ll})+\beta\left[\kappa U^{Aut}(y_l)+(1-\kappa)U^{Aut}(y_h)\right]$$

for the low-income agents and by

$$U^{Aut}(y_h,k_h)=(1-\beta)(u(y_h)-\delta^h_{hh})+\beta\left[(1-\kappa)\frac{\tilde{\omega}^{k_h}_{k_l}+\tilde{\omega}^{k_h}_{k_h}}{2}+\kappa U^{Aut}(y_h)\right]$$
$$U^{Aut}(y_h,k_h)=(1-\beta)(u(y_h)-\delta^l_{hh})+\beta\left[(1-\kappa)\frac{\tilde{\omega}^{k_h}_{k_l}+\tilde{\omega}^{k_h}_{k_h}}{2}+\kappa U^{Aut}(y_h)\right]$$
$$U^{Aut}(y_h,k_l)=(1-\beta)(u(y_h)-\delta^h_{hl})+\beta\left[\kappa\frac{\tilde{\omega}^{k_l}_{k_l}+\tilde{\omega}^{k_l}_{k_h}}{2}+(1-\kappa)U^{Aut}(y_h)\right]$$
$$U^{Aut}(y_h,k_l)=(1-\beta)(u(y_h)-\delta^l_{hl})+\beta\left[\kappa\frac{\tilde{\omega}^{k_l}_{k_l}+\tilde{\omega}^{k_l}_{k_h}}{2}+(1-\kappa)U^{Aut}(y_h)\right],$$

for the high-income agents, where $U^{Aut}(y_h)=0.5(U^{Aut}(y_h,k_h,n_h)+U^{Aut}(y_h,k_l,n_h))$, $U^{Aut}(y_h),\tilde{w}$ are defined correspondingly.

Outside options do not depend on previous period signals, thus, $\delta^h_{hh}=\delta^l_{hh}=\delta_{hh}$ and $\delta^h_{hl}=\delta^l_{hl}=$

δ_{hl}. The marginal utility of low-income agents before the the transfer and the Pareto weight are identical for each combination of past and current public signal. Thus, low-income agents should optimally receive the same transfer, $\delta_{li}^{j} = \epsilon/[4(1-\beta)]$ for all i, j, where the sum of transfers to low income agents is normalized to $\epsilon/(1-\beta)$. The utility of high-income agents is equal to their outside option and therefore independent from the particular transfer scheme. The transfers of high-income agents can be directly derived from their outside option value, reflecting their expected future gain from transferring in the current period. It follows that the transfer scheme can be summarized by

$$\delta_l \equiv \sum_{i,j} \delta_{li}^{j} = \frac{\epsilon}{1-\beta} \quad \delta_{hh} = \beta\epsilon\frac{(1-\kappa)}{4(1-\beta)} \quad \delta_{hl} = \beta\epsilon\frac{\kappa}{4(1-\beta)}.$$

The distribution $\hat{\Phi}$ requires the following increase in resources

$$\begin{aligned}\Delta &= \pi_h(1-\pi_h)c'[u(y_l)]\delta_l/4 - \frac{\pi_h}{2}c'[u(y_h)](\delta_{hh}+\delta_{hl}) \\ &= \frac{\pi_h(1-\pi_h)\epsilon}{4(1-\beta)}\left[\frac{1}{u'(y_l)} - \frac{\beta}{2(1-\pi_h)u'(y_h)}\right].\end{aligned} \tag{27}$$

If

$$\beta < \left[\frac{(1-\pi_h)u'(y_h)}{u'(y_l)}2\right],$$

the constructed allocation $\hat{\Phi}$ violates resource feasibility. Using $\pi_h = 1/2$ results in the expression stated in the proposition.

A.4 Proof of Proposition 4

Proof. With partial risk sharing, the two participation constraints of high-income agents are binding but the correspond constraint for low-income agents are slack.[10] They are given by the following expressions

$$\begin{aligned}F(c_h^h, c_l^h) \equiv{}& (1-\beta)u(c_h^h) + \beta(1-\beta)\kappa V_{rs}^h + \beta(1-\beta)(1-\kappa)V_{rs}^l + \beta^2 V_{rs} \\ &- (1-\beta)u(y_h) - \beta(1-\beta)\left(\kappa u(y_h) + (1-\kappa)u(y_l)\right) - \beta^2 V_{out} = 0,\end{aligned}$$

[10] A proof for this result can be found for example in Lepetyuk and Stoltenberg (2013). In history-dependent arrangements also participation constraints of low-income agents are occasionally binding.

$$G(c_h^h, c_l^h) \equiv (1-\beta)u(c_l^h) + \beta(1-\beta)(1-\kappa)V_{rs}^h + \beta(1-\beta)\kappa V_{rs}^l + \beta^2 V_{rs}$$
$$- u(y_h) - \beta(1-\beta)\left((1-\kappa)\, u(y_h) + \kappa u(y_l)\right) - \beta^2 V_{out} = 0,$$

1. The derivative of the conditional mean of consumption of high-income agents is given by

$$\frac{1}{2}\left(\frac{\partial c_h^h}{\partial \kappa} + \frac{\partial c_l^h}{\partial \kappa}\right) = \frac{x}{2}\left(\frac{F_{c_l^h} + G_{c_l^h}}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}} - \frac{F_{c_h^h} + G_{c_h^h}}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}}\right),$$

with

$$x \equiv \beta(1-\beta)(u(y_h) - V_{rs}^h - u(y_l) + V_{rs}^l) \geq 0.$$

At the optimal memoryless allocation, we have that

$$F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h} > 0$$

which follows from autarky not being the optimal memoryless allocation (see Lemma 1 in Section A.5). Further, the partial derivatives are

$$F_{c_h^h} = (1-\beta)\left[u'(c_h^h) + \beta\frac{\kappa}{2}u'(c_h^h) - \beta\frac{1-\kappa}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_h^h) - u'(c^l)\right]$$
$$F_{c_l^h} = (1-\beta)\left[\beta\frac{\kappa}{2}u'(c_l^h) - \beta\frac{1-\kappa}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_l^h) - u'(c^l)\right]$$
$$G_{c_l^h} = (1-\beta)\left[u'(c_l^h) + \beta\frac{1-\kappa}{2}u'(c_l^h) - \beta\frac{\kappa}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_l^h) - u'(c^l)\right]$$
$$G_{c_h^h} = (1-\beta)\left[\beta\frac{1-\kappa}{2}u'(c_h^h) - \beta\frac{\kappa}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_h^h) - u'(c^l)\right].$$

After some steps of tedious but straightforward algebra, one gets

$$\frac{1}{2}\left(\frac{\partial c_h^h}{\partial \kappa} + \frac{\partial c_l^h}{\partial \kappa}\right) = \frac{x}{2}(2-\beta)\frac{u'(c_l^h) - u'(c_h^h)}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}} \geq 0$$

because $c_h^h \geq c_l^h$ follows from a higher outside option value of agents with a good public signal

$$u(y_h) + \beta(1-\beta)\left(\kappa u(y_h) + (1-\kappa)\, u(y_l)\right) + \beta^2 V_{out} \geq$$
$$u(y_h) + \beta(1-\beta)\left((1-\kappa)\, u(y_h) + \kappa u(y_l)\right) + \beta^2 V_{out}. \quad (28)$$

Non-binding participation constraints for low-income agents result in equal consumption of agents with a good and bad public signal. Resource feasibility then implies that conditional consumption of low-income agents decreases in signal precision.

2. The conditional standard deviation of low-income agents is not affected because consumption of these agents is equalized in optimal memoryless allocations. The variance of consumption of high-income agents is

$$\text{var}(c|y^h) = \frac{1}{2}\left[(c_h^h - \bar{c}^h)^2 + (c_l^h - \bar{c}^h)^2\right], \quad (29)$$

with

$$\bar{c}^h = \frac{c_h^h + c_l^h}{2}.$$

From the first order conditions at the optimal memoryless allocation, we get

$$F_{c_l^h} + G_{c_l^h} > 0 \quad F_{c_h^h} + G_{c_h^h} > 0.$$

Thus, the derivatives of high-income agents' consumption with respect to signal precision satisfy

$$\frac{\partial c_h^h}{\partial \kappa} \geq 0 \quad \frac{\partial c_l^h}{\partial \kappa} \leq 0.$$

From Part 1, we know that the conditional mean of high-income agents is increasing in signal precision but less than c_h^h because c_l^h decreases. Thus, both terms in (29) increase such that the conditional standard deviation of high-income agents increases.

3. The unconditional variance of consumption is

$$\text{var}(c) = \frac{1}{4}\left[(c_h^h - \bar{y})^2 + (c_l^h - \bar{y})^2 + 2(c^l - \bar{y})^2\right].$$

From Part 1, we get that $c^l < \bar{y}$ is decreasing in signal precision, where the inequality follows from perfect risk sharing not being feasible. From Part 2, $c_h^h > \bar{y}$ is increasing while $c_l^h > \bar{y}$ is decreasing in signal precision but from Part 1, we get that the conditional mean of consumption of high-income agents increases in signal precision. This implies that $\mathrm{d}\, c_h^h \geq -\mathrm{d}\, c_l^h$. Thus, all terms on the right-hand side increase in signal precision and the unconditional standard deviation of consumption increases.

■

A.5 Lemma on autarky with memoryless allocations

In the proof of Proposition 4, we make use of the following property.

Lemma 1 *Assume that participation constraints of high-income agents are binding. If*

$$(2-\beta)u'(y_h) - \beta u'(y_l) < 0$$

then autarky is not the optimal memoryless allocation.

Proof. The optimal memoryless arrangement can be analyzed as a fixed-point problem expressed in terms of the period value of the arrangement.

The fixed-point problem is constructed as follows. Let W be the unconditional expected value of an arrangement before any signal has realized. We restrict attention to $W \in [V_{out}, \bar{V}_{rs})$ because per assumption participation constraints for high-income households are binding. The binding participation constraints are given by the following

$$\begin{aligned} & u(c_h^h) + \frac{\beta \left\{ \kappa\nu [u(c_h^h) + u(c_l^h)]/2 + (1-\kappa)(1-\nu) u(2\bar{y} - (c_h^h + c_l^h)/2) \right\}}{\kappa\nu + (1-\kappa)(1-\nu)} \\ & = u(y_h) + \frac{\beta \left[\kappa\nu u(y_h) + (1-\kappa)(1-\nu) u(y_l) \right]}{\kappa\nu + (1-\kappa)(1-\nu)} + \frac{\beta^2}{1-\beta}(V_{out} - W), \end{aligned} \tag{30}$$

$$\begin{aligned} & u(c_l^h) + \frac{\beta \left\{ (1-\kappa)\nu [u(c_h^h) + u(c_l^h)]/2 + \kappa(1-\nu) u(2\bar{y} - (c_h^h + c_l^h)/2) \right\}}{(1-\kappa)\nu + \kappa(1-\nu)} \\ & = u(y_h) + \frac{\beta \left[(1-\kappa)\nu u(y_h) + \kappa(1-\nu) u(y_l) \right]}{(1-\kappa)\nu + \kappa(1-\nu)} + \frac{\beta^2}{1-\beta}(V_{out} - W), \end{aligned} \tag{31}$$

and resource feasibility is used. The objective function of the problem to compute the optimal memoryless arrangement is given by the following expression

$$V_{rs}(W) \equiv \frac{1}{4}\left[u(c_h^h(W)) + u(c_l^h(W)) + 2u(2\bar{y} - (c_h^h(W) + c_l^h(W))/2)\right].$$

The optimal memoryless arrangement should necessary solve the fixed-point problem $W = V_{rs}(W)$. We will show that $V_{rs}(W)$ is strictly increasing. $V(W)$ is also strictly concave, therefore there exist at most two solutions to the fixed-point problem.

From the participation constraints (30) and (31), the derivative of $V(W)$ is given by

$$V'_{rs}(W) = \frac{1}{4}\left[(u'(c_h^h) - u'(c_l))\frac{\partial c_h^h}{\partial W} + (u'(c_l^h) - u'(c_l))\frac{\partial c_l^h}{\partial W}\right]$$

which is strictly increasing in W because perfect risk sharing is not constrained feasible which implies that $\partial c_h^h/\partial W$ and $\partial c_l^h/\partial W$ are negative and $c_h^h, c_l^h \neq \bar{y}$.

By construction, one solution to the fixed-point problem is V_{out}. The concavity of $V_{rs}(W)$ implies that the derivative of $V_{rs}(W)$ at V_{out} is higher than at any partial risk-sharing allocation and there are at most two solutions. Therefore, when autarky is not the optimal memoryless arrangement the derivative of $V'_{rs}(w)$ at V_{out} must larger than 1

$$V'_{rs}(W) = \frac{1}{4}\left[(u'(y_h) - u'(y_l)\right]\left(\frac{\partial c_h^h}{\partial W} + \frac{\partial c_l^h}{\partial W}\right)\Bigg|_{\{c_i^j\}=\{y_j\}} > 1$$

The two derivatives are

$$\frac{\partial c_h^h}{\partial W} = -\frac{\begin{vmatrix} \beta^2 & P_{c_l^h} \\ \beta^2 & Q_{c_l^h} \end{vmatrix}}{\begin{vmatrix} P_{c_h^h} & P_{c_l^h} \\ Q_{c_h^h} & Q_{c_l^h} \end{vmatrix}}, \quad \frac{\partial c_l^h}{\partial W} = -\frac{\begin{vmatrix} P_{c_h^h} & \beta^2 \\ Q_{c_h^h} & \beta^2 \end{vmatrix}}{\begin{vmatrix} P_{c_h^h} & P_{c_l^h} \\ Q_{c_h^h} & Q_{c_l^h} \end{vmatrix}},$$

with the auxiliary derivatives P, Q as the partial derivatives of the binding participation constraints

(30) and (31) evaluated at the autarky allocation given by

$$
\begin{aligned}
P_{c_h^h} &= (1-\beta)[u'(y_h) + \frac{\beta}{2}\kappa u'(y_h) - \frac{\beta}{2}(1-\kappa)u'(y_l)] \\
P_{c_l^h} &= P_{c_h^h} - (1-\beta)u'(y_h) \\
Q_{c_l^h} &= (1-\beta)[u'(y_h) + \frac{\beta}{2}(1-\kappa)u'(y_h) - \frac{\beta}{2}\kappa u'(y_l)] \\
Q_{c_h^h} &= Q_{c_l^h} - (1-\beta)u'(y_h).
\end{aligned}
$$

Using these expression, the sum of the partial derivatives with respect to W evaluated at $\{c_i^j\} = \{y_j\}$ is given by

$$
\begin{aligned}
\left(\frac{\partial c_h^h}{\partial W} + \frac{\partial c_l^h}{\partial W}\right) &= -\frac{2\beta^2(1-\beta)u'(y_h)}{(1-\beta)u'(y_h)[P_{c_h^h} + Q_{c_l^h} - (1-\beta)u'(y_h)]} \\
&= \frac{-4\beta}{(1-\beta)\left[u'(y_h)\left(\frac{2}{\beta}+1\right) - 2u'(y_l)\right]}.
\end{aligned}
$$

Using this expression in $V_{rs}'(W)$ and collecting terms eventually results in

$$
u'(y_h)(2-\beta) > \beta u'(y_l).
$$

Under this condition, the optimal memoryless arrangement is not the outside option and thus characterized by risk sharing. It implies that at the optimal allocation $V'(W) < 1$ which leads to

$$
F_{c_h^h} G_{c_l^h} - F_{c_l^h} G_{c_h^h} > 0
$$

which is used in the proof of Proposition 4. ■

A.6 Details on the joint distribution of income and signals

In this subsection, we explain how to derive the formulas (2) and (3) stated in the main text. Further, we explain the logic behind the assumption that the stochastic process for signals shares the transition probabilities with the process for income.

A.6.1 Derivation of the formulas on the joint distribution of income and signals

We start with the derivation of the conditional probability of income. Using the general formula for calculating conditional probabilities, we receive

$$\pi\left(y' = y_j | k = y_m, y = y_i\right) = \frac{\pi\left(y' = y_j, k = y_m, y = y_i\right)}{\pi\left(k = y_m, y = y_i\right)}.$$

The conditional probability of income can be simplified using the identity

$$\sum_{z=1}^{N} \pi\left(y' = y_z | k = y_m, y = y_i\right) = 1$$

to replace the denominator with the following expression

$$\pi\left(k = y_m, y = y_i\right) = \sum_{z=1}^{N} \pi\left(y' = y_z, k = y_m, y = y_i\right).$$

The joint probability in the numerator is

$$\pi\left(y' = y_j, k = y_m, y = y_i\right) = \pi_{ij} \kappa^{\mathbf{1}_{m=j}} \left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=j}},$$

where π_{ij} is the Markov transition probability for moving from income i to income z. For all income states that are not indicated by the signal, $j \neq m$, we assume here that their probability of occurrence conditional on the signal is identical and therefore equals $(1-\kappa)/(N-1)$. For the conditional probability of income, the general formula can then be written as

$$\pi\left(y' = y_j | k = y_m, y = y_i\right) = \frac{\pi_{ij} \kappa^{\mathbf{1}_{m=j}} \left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=j}}}{\sum_{z=1}^{N} \pi_{iz} \kappa^{\mathbf{1}_{m=z}} \left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=z}}} \qquad (32)$$

which resembles (2) in the main text. For example, with two equally likely persistent income states, the conditional probability of receiving a low income y_l in the future conditional on a high signal $k = y_h$ and a low income today is given according to (32) by

$$\pi\left(y' = y_l | k = y_h, y = y_l\right) = \frac{(1-\kappa)\pi_{11}}{(1-\kappa)\pi_{11} + (1-\pi_{11})\kappa}.$$

The joint transition probability $\pi(s'|s) = \pi\left(y', k'|k, y\right)$ can be computed by combining the conditional probability of income with an assumption on the signal process. With signals following an exogenous first-order Markov process, the conditional probability $\pi\left(y', k'|k, y\right)$ is given by

$$\pi\left(y' = y_j, k' = y_l | k = y_m, y = y_i\right) = \pi_{ml} \frac{\pi_{ij} \kappa^{\mathbf{1}_{m=j}} \left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=j}}}{\sum_{z=1}^{N} \pi_{iz} \kappa^{\mathbf{1}_{m=z}} \left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=z}}} \quad \forall k', \tag{33}$$

where compared to (3), we used $\pi(k' = y_l | k = y_m) = \pi_{ml}$ because the signal process is characterized by the same transition probabilities as income. In the following, we argue why we choose signals that share the transition probabilities with income.

A.6.2 Consistency requirements with exogenous signals

We assumed that the signal realizations share the transition probabilities with the stochastic income process. In the following, we argue why we make this assumption by comparing implications of this assumption to alternative stochastic processes for signals. In our environment, income is the fundamental uncertainty and signals provide additional information on future realizations of income without affecting the realizations of income themselves. More formally, the later implies that the joint distribution of income and signals should satisfy the following two consistency requirements.

Consistency Requirement I: The marginal distribution of the joint invariant distribution $\pi(s) = \pi(y, k)$ equals the invariant distribution of income $\pi(y)$ i.e.

$$\hat{\pi}(y) = \sum_{k \in Y} \pi(y, k) \doteq \pi(y)$$

Consistency Requirement II: The conditional distribution of income $\pi(y'|y)$ follows from integrating over the signals

$$\hat{\pi}(y'|y) = \sum_{k \in Y} \pi(y'|y, k) \pi(k|y) \doteq \pi(y'|y).$$

In the following proposition, we show that when signals follow the same stochastic process as income, the two requirements are satisfied. If signals were to follow a different process then at least one of the requirements is violated. For the analytical results, we consider an income process with two values y_l and y_h and a symmetric transition between income states. The transition matrix for these two income states is given as P=

$$\begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}$$

where rows represent the present income state and columns represent the future income states. For $p = 0.5$, income states is i.i.d.

Proposition 5 *Consider a Markov income process with transition matrix P.*

(i) If signals follow the same stochastic process as income then both consistency requirements are satisfied.

(ii) Consider a Markov process for signals with transition matrix $\tilde{P}$

$$\tilde{P} = \begin{bmatrix} \tilde{p} & 1-\tilde{p} \\ 1-\tilde{p} & \tilde{p} \end{bmatrix}$$

and $0 < \tilde{p} < 1$, $\tilde{p} \neq p$. Then Consistency Requirement II is violated.

Proof.

(i) When signals follow the same transition probabilities as income, the transition probabilities of s can be computed using (3) and are then summarized in the transition matrix P_s. For example, the probability of a low income and a low signals conditional on a low income and signal is

$$\pi\left(y' = y_l, k' = y_l | k = y_l, y = y_l\right) = p\frac{\kappa p}{(1-\kappa)(1-p) + p\kappa}.$$

The unique stationary distribution corresponding to the transition matrix P_s is given by

$$\pi(y,k) = \begin{bmatrix} \pi(y_l,k_l) \\ \pi(y_l,k_h) \\ \pi(y_h,k_l) \\ \pi(y_h,k_h) \end{bmatrix} = \begin{bmatrix} \kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2} \\ \frac{\kappa}{2} + \frac{p}{2} - \kappa p \\ \frac{\kappa}{2} + \frac{p}{2} - \kappa p \\ \kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2} \end{bmatrix}$$

Adding the first two and last two rows show that Consistency Requirement I is satisfied. Further, the probabilities of signals conditional on income can be computed from the invariant distribution. For example, the probability of a low signal conditional on a low income can be computed as

$$\pi(k = y_l | y = y_l) = \frac{\kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2}}{\kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2} + \frac{\kappa}{2} + \frac{p}{2} - \kappa p} = 2\kappa p - \kappa - p + 1.$$

To check for the Consistency Requirement II, we consider present income $y = y_l$ and future income $y' = y_l$ (the other transitions can be computed in the same way and are omitted here)

$$\begin{aligned}
\hat{\pi}(y' = y_l | y = y_l) &= \sum_{k \in Y} \pi(y' = y_l | y = y_l, k)\pi(k | y = y_l) \\
&= \pi(y' = y_l | y = y_l, k = y_l)\pi(k = y_l | y = y_l) + \pi(y' = y_l | y = y_l, k = y_h)\pi(k = y_h | y = y_l) \\
&= \frac{\kappa p}{\kappa p + (1-\kappa)(1-p)}(2\kappa p - \kappa - p + 1) + \frac{p(1-\kappa)}{\kappa(1-p) + p(1-\kappa)}(\kappa + p - 2\kappa p) \\
&= p
\end{aligned}$$

which is also satisfied. From the other side, for the transition from low income today to low income in the future, Requirement II calls for

$$p \doteq \pi(y' = y_l | y = y_l, k = y_l)\hat{\pi}(k = y_l | y = y_l) + \pi(y' = y_l | y = y_l, k = y_h)[1 - \hat{\pi}(k = y_l | y = y_l)],$$

which has as unique solution $\hat{\pi}(k = y_l | y = y_l) = 2\kappa p - \kappa - p + 1$ which completes the proof of part (i).

(ii) The general symmetric transition matrix for signals $\tilde{P}$ results in a joint transition matrix

	Consistency requirements	
	I, $\max(\lvert\widehat{\pi}(y)-\pi(y)\rvert)$	II, $\max(\lvert\widehat{\pi}(y'\lvert y)-\pi(y'\lvert y)\rvert)$
i.i.d. signals	0.0115	0.1620
persistent signals	$3.70e-16$	$2.11e-16$

Table 5: Consistency requirement results with persistent income

for signals and income $\tilde{P}_s$ and in a unique invariant distribution for income and signals $\tilde{\pi}(y,k)$ with a unique conditional probability $\tilde{\pi}(k=y_l|y=y_l)$. If an only if $\tilde{p}=p$, it is $\tilde{\pi}(k=y_l|y=y_l)=\hat{\pi}(k=y_l|y=y_l)=2\kappa p-\kappa-p+1$. Thus, Requirement II is violated for $\tilde{p}\neq p$. Requirement I is satisfied because $\sum_k \tilde{\pi}(y_l,k)=1/2=\sum_k \tilde{\pi}(y_h,k)$ for any $0<\tilde{p}<1$.

■

As an immediate implication of the proposition, i.i.d. signals violate Requirement II when income is persistent. When income has more than two states, i.i.d. signals violate not only the second requirement. In Table 5, we also compare both signal processes using the income process employed for computing the quantitative results in Section 4 for $\kappa=0.99$ as an extreme case. As displayed in the first row of the table, i.i.d. signals fail both consistency requirements. The inconsistency following from i.i.d. signals is not negligible. On average, i.i.d. signals imply a perceived income transition that differs from the true transition by 16 percent. Persistent signals with the same persistence as income continue to satisfy both requirements (see the second row).[11]

A.7 Numerical algorithm: limited commitment endowment economy

To solve the limited commitment model with signals, we follow the policy function algorithm proposed by Coleman (1990) and extended for these models by Krueger and Perri (2011). Given a discount factor β and state (w,s), we search for the optimal piecewise linear functions $\{w'_{s'}(w,s)\}_{s'\in S}$ and $h(w,s)$. We apply the Ridder algorithm and check for convergence on the discount factor. The algorithm can be summarized in the following steps

Step 1: Solve for the autarky values and get the maximum and minimum values of the outside option $(\underline{w},\overline{w})$.

[11] We use the method proposed by Tauchen and Hussey (1991) to approximate the income process. This method implies a symmetric transition matrix.

Step 2: Create the grid for the promised values $w_{grid} = \{\underline{w}, \cdots, \overline{w}\}$.

Step 3: Guess an initial value for $V_w^0(w, s)$, where the subscript refers to differentiation with respect to w.

Step 4: Use the first order conditions (10) and (11) and the promise keeping constraint (8) to solve for $h(w, s)$ and $\{w'_{s'}(w, s)\}$ for each $(w, s) \in w_{grid} \times S$[12]. The solution at each grid point involves two steps.

- First check which of the participation constraints are binding. The ones which are binding, replace them with the corresponding autarky value. Solve for the remaining values. For N states, then we have to solve for each grid point $N + 1$ equations. We can solve this as there are N first order conditions and one promise keeping constraint. If $M \leq N$ participation constraints are binding then we drop those variables and the corresponding first order conditions.
- After solving, again check whether any $w'_{s'}(w, s)$ violates the participation constraint in state s'. If yes, then replace the corresponding promise with the autarky value.

Step 5: Update $V_w^0(w, s)$ to $V_w^1(w, s)$ using the envelope condition (12).

Step 6: Check whether convergence in $h(w, s), \{w'_{s'}(w, s)\}$ and $V_w(w, s)$ is achieved. Otherwise go back to Step 4 with the updated value of $V_w^1(w, s)$

In the next step, use policy functions $h(w, s)$ and $w'_{s'}(w, s)$ (that are piece-wise linear in w) to compute the invariant distribution in the following manner. For each grid point (w, s) find $\underline{w}_{s'}(w, s)$ and $\overline{w}_{s'}(w, s)$ such that

$$\underline{w}_{s'}(w, s) = \max\{w \in w_{grid} | w \leq w'_{s'}(w, s)\}$$
$$\overline{w}_{s'}(w, s) = \min\{w \in w_{grid} | w > w'_{s'}(w, s)\}$$

Then find $\alpha_{s'}(w, s)$ which solves the equation

$$\alpha_{s'}(w, s)\underline{w}_{s'}(w, s) + (1 - \alpha_{s'}(w, s))\overline{w}_{s'}(w, s) = w'_{s'}(w, s)$$

[12] Note that $h(w, s)$ and $\{w'_{s'}(w, s)\}$ are not restricted to be on the w grid.

Using $\alpha_{s'}(w,s)$ define the Markov transition matrix $Q : (W \times S) \times (W \times S) \to [0,1]$ as

$$Q[(w,s),(w',s')] = \begin{cases} \pi(s'|s)\alpha_{s'}(w,s) & \text{if} \quad w' = \underline{w}_{s'}(w,s) \\ \pi(s'|s)(1-\alpha_{s'}(w,s)) & \text{if} \quad w' = \overline{w}_{s'}(w,s) \\ 0 & \text{otherwise} \end{cases}$$

The invariant distribution $\Phi_{w,s}$ is then computed as the normalized eigenvector of Q corresponding to the unit eigenvalue. Using the invariant distribution compute the excess demand

$$d(\beta) = \int C(w,s)\,\mathrm{d}\,\Phi_{w,s} - \int y\,\mathrm{d}\,\Pi(y)$$

and check whether it is satisfied. If not, decrease β if $d(\beta)$ is in surplus and increase β if it is in deficit, and go back to Step 4. We use a Ridder algorithm until convergence on the discount factor is achieved and excess demand equals zero.

A.8 Numerical algorithm: limited commitment production economy

Given initial wealth a, state $s = (y,k)$, and an interest rate R, households' problem can be written recursively as

$$V(a,s) \quad = \quad \max_{c,\{a'\}} \left[(1-\beta)u(c(a,s)) + \beta \sum_{s'} \pi(s'|s)V'(a'(a,s;s'),s')\right]$$

subject to a budget and a borrowing constraint

$$c + \sum_{s'} \frac{\pi(s'|s)a'(a,s;s')}{R} \quad \leq \quad y + a \tag{34}$$

$$a'(a,s;s') \quad \geq \quad A(s'), \quad \forall s'. \tag{35}$$

The borrowing limits satisfy the following equations

$$U^{Aut}(s') = V'[A(s'),s'], \quad \forall s'. \tag{36}$$

The first order conditions are

$$u'[c(a,s)](1-\beta) = \lambda = V_a(a,y) \tag{37}$$

$$\beta V_a'[a'(a,s;s'),s'] \leq \frac{u'[c(a,s)](1-\beta)}{R}, \quad \forall s', \tag{38}$$

where $V_a'[a'(a,s;s'),s']$ denotes the derivative of the value function with respect to $a'(a,s;s')$. Consider N income states such that $s \in S = (s_1, s_2, ...s_{N^2})$. Consider a grid for a. Start with a guess of the value function V_0 and for the derivative $V_{a,0}$. From the guess of the value function, back out the state-dependent borrowing limits $A_0(s')$ from (36).

1. For each pair a, s, solve for the policy functions $c_0(a,s), \{a_0'(a,s;s')\}$ using the N^2+1 first order conditions (38) and (34). Start with the strict equality for all s' and solve. Check borrowing constraints. If not satisfied in some state s', set $a_0'(a,s;s') = A_0(s')$ and solve again for $c_0(a,s)$ and the remaining $a_0'(a,s;s')$ until no borrowing constraint is violated.

2. Update the derivative of the value function with respect to a using the envelope condition and the policy function for consumption

$$V_{a,1}(a,s) = u'[c_0(a,s)](1-\beta)$$

3. Update the value function according to the Bellman equation to receive V_1

$$V_1(a,s) = (1-\beta)u[c_0(a,s)] + \beta \sum_{s'} \pi(s'|s) V_0[a_0'(a,s;s'),s']$$

4. Continue until convergence in the policy functions, the derivative of the value function and in the value function $V_n(a,s) = V_{n+1}(a,s) = V(a,s)$ is achieved.

5. Then update the borrowing limits solving the following equation for A_1

$$V[A_1(s'),s)] = U^{aut}(s').$$

6. Continue until convergence in the policy functions, in the value function (and its derivative) and in the borrowing limits is achieved.

The computation of the invariant distribution $\Phi_{a,s}$ follows the same steps as in the endowment economy. The excess demand on the goods market now reads

$$d_K(\beta) = \int c(a,s)\,\mathrm{d}\,\Phi_{a,s} + K' - K(1-\delta) - AF(L,K).$$