Non-Exclusive Financial Advice*

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Abstract

We study a model of financial advice where investors rely on a financial expert (the advisor) to make their asset allocation choices. There is only one source of risk and the advisor is privately informed about the volatility of the return of the risky asset. Moreover, the advisor's preferences are misaligned with those of his uninformed clients, and this conflict of interests cannot be solved by means of state-contingent monetary transfers. In equilibrium, investors delegate the investment decision to the financial advisor. However, they impose restrictions on the advisor's choices. These restrictions take the form of a cap or a floor on the amount invested in the risky asset. The precise form of *partial* delegation that emerges depends on whether financial advice is exclusive or not, and in the case of non-exclusive advice, on whether the common advisor perceives investors' asset allocations as complements or as substitutes. We also analyze the implications of non-exclusivity in financial advice on investment behavior and investors' expected utility.

Keywords: Delegated Portfolio Management, Financial Advice, Non-Exclusivity

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1. Introduction

The importance of money managers as a vehicle to provide valuable information to investors has been recognized by the finance literature (e.g., Inderst and Ottaviani, 2012a-2012b). In most developed countries, a substantial portion of financial wealth is not managed directly by savers, but rather by specialized intermediaries. This is particularly true in the case of small and less knowledgeable investors, whose portfolio choices typically rely on the superior information provided by their financial advisors (banks).[1]

The importance of financial advisors has also been highlighted in recent surveys. A survey conducted by Hung et al. (2008) reveals that 73 percent of all US retail investors consult a financial advisor before purchasing shares. In a large online survey among recent purchasers of investment products in the EU, Chater et al. (2010) found that nearly 80 percent made their purchases through an intermediary, with 58 percent of them claiming that an advisor influenced their choice. These studies highlight an increasing trend towards more reliance on financial advice in spite of an easier access to financial markets (for example through the internet). Yet, there is some evidence that active management underperforms relative to a passive benchmark (e.g., Chevalier and Ellison, 1997; Gruber, 1996; and Malkiel, 1995). One possible reason is that money managers typically have objectives that are not fully aligned with those of their clients. This divergence of interests creates scope for opportunistic behavior that may distort financial advice and induce households to make investment choices that perform worst than expected.[2]

In this paper we study the interaction between small investors and their financial advisor when the advisor has private information about the riskiness of some of the existing assets *and* the objectives of the investors and those of the advisor are not identical. We do so in a particular environment. First, we assume that monetary transfers between investors and the financial advisor are not possible. This means that investors cannot provide incentives to the advisor by means of money transfers. Second, we allow for non-exclusive relationships between investors and the financial advisor. While less appealing in the case of institutional investors, whose size may come along with bargaining power *vis-à-vis* money managers, both of these assumptions seem more compelling in the case of small investors (e.g., households). When making their investment decisions, households are usually advised by their banks' employees, whose services are typically non-exclusive and do not require additional costs over and above the (fixed) fees required to open a deposit account. Finally, we assume that investors can choose and commit to an investment rule (a mechanism) which specifies their portfolio allocation as a function of the information reported by the advisor. We characterize the optimal rule from the investor's perspective when the relationship with the advisor is exclusive. We also characterize the investment rules chosen

[1] The lack of basic financial education among households has been documented in many studies (e.g., Lusardi and Mitchell,2007).

[2] Basak et al. (2003), for instance, show that the costs of misaligned incentives resulting from delegated portfolio management are potentially significant when the managers' and investors' attitudes towards risk differ substantially.

by investors in equilibrium when investors rely on a common financial advisor. In particular, we analyze how the externalities investors impose on one another through the preferences of their common advisor shape their investment rules and overall investment behavior.

More specifically, we study a simple model of non-exclusive financial advice where an expert advises two identical investors who desire to invest their money into a risky asset, but do not have enough knowledge to do it personally. The expert has private information about the riskiness of this asset (the state of the world). Each investor chooses an asset allocation rule that maps the advisor's report about the state of the world into a portfolio choice. Focusing on the class of continuous asset allocation rules, we characterize the equilibria of the resulting (intrinsic) common agency game and determine under which conditions these outcomes feature 'delegated portfolio management' and when, instead, clients prefer to enforce rigid investment rules that are unresponsive to the information reported by the advisor. We assume that investors are risk averse with mean-variance utility functions, while the preferences of the money manager are represented by a loss function that depends on the (investment) choices of both clients. Specifically, the advisor's ideal investment choices differ from those of his clients: a conflict of interests that creates an incentive to misreport the state of the world. This conflict of interests requires investors to choose asset allocations that are incentive feasible, which in turn leads to equilibria that are distorted away from the first-best.

Although we will develop most of our formal analysis under the hypothesis that investors choose their equilibrium portfolio allocation by committing to direct mechanisms — i.e., investment choices contingent on the information reported by the advisor — the same outcomes can be implemented by extremely simple delegation rules that only need to specify the range of investment choices that investors are willing to accept, and leave otherwise to the advisor full control over the actual portfolio composition in spite of his misalignment of preferences: a delegated portfolio management result.

We start the analysis by considering the case where the advisor deals with only one client — i.e., the case of exclusive financial advice. While this case is interesting *per se*, it is also used as a benchmark against which we compare the outcome obtained when financial advice is non-exclusive. We focus on the case where the advisor wants the investor to overinvest in the risky asset relative to the investor's optimal allocation. In this case, the advisor has an incentive to claim that the riskiness of this asset is lower than its actual value. The investor takes into account this incentive of the advisor when designing the optimal investment rule. We show that the optimal rule consists of delegating the investment decision to the advisor, but imposing a cap on the amount the advisor can allocate to the risky asset. The delegation of the investment decision allows the investor to incorporate the relevant private information of the advisor in the composition of his portfolio. The imposition of a cap allows the investor to deal with the fact that the advisor's preferences differ from his preferences. The optimal cap balances the benefit from using the advisor's private information on the riskiness of the risky asset and the cost stemming from the misalignment of incentives. We interpret a higher cap as more delegation.

We also show that the more aligned are incentives or the higher the uncertainty about riskiness of the risky asset, the more the investor delegates his portfolio decision on the advisor.

We next analyze the case in which investors rely on a common advisor when deciding their portfolio allocations – i.e., the case of non-exclusive financial advice. In this case, the advisor's payoff may be non-separable with respect to his clients' portfolio choices. This means that the portfolio choice of one client may affect the advisor's ideal investment choice of the other client (and vice versa). When this is the case, there are indirect externalities between the clients that originate from the preference structure of their common advisor. We show that both the sign and the magnitude of these externalities are key to determine the equilibrium portfolio choices. For simplicity, we consider two alternative scenarios: one where the expert always perceives his clients' portfolio choices as *substitutes*, and the other where these choices are always perceived as *complements*. Investment choices are perceived as substitutes when the advisor's incentive to induce a higher risk exposure by one client weakens as the other client's exposure increases. In contrast, investment choices are perceived as complements when the advisor's incentive to induce a higher risk exposure by one client strengthens when the exposure of the other client increases too.

In reality, whether portfolio choices are perceived as substitutes or as complements by a financial advisor depends on her indirect utility function, which in turn depends on her (original) preferences and the incentive scheme to which she is exposed by her employer (e.g., a bank). We take this indirect utility function as given and characterize the investment rules investors choose in equilibrium for different types of such utility function. One can think, however, that substitutability may either be driven by regulatory constraints that impose binding liquidity requirements to banks, so that money managers may be concerned with maintaining a certain amount of money on their clients' deposit accounts; or it could reflect the expert's reputational concerns that arise when the bank's top management is sensible to clients' dissatisfaction — e.g., 'misselling' costs that may arise from litigation procedures (perhaps due to class actions) or costs of foregone customers (see Calcagno and Monticone, 2013, and Inderst and Ottaviani, 2009). Instead, complementarity may result from sales commissions paid to the bank by the provider of the financial product that reward not only the achievement of investment targets on each single client, but that also award premia that are increasing with the size of the clientele gathered by the bank — e.g., when funds reward the achievement of clients' targets. Alternatively, it could be due to the 'empire building' desire of the advisor — e.g., a large portfolio of clients may provide him a stronger bargaining position *vis-à-vis* the bank's top management.

We show that when the financial advisor perceives investors' portfolio choices as complements she has an incentive to induce each investor to take *even more* risk than in the case of exclusive financial advice. As a result, investors trust less the financial advisor and delegate less.[3] That

[3] Notice that in our model trust in the financial advisor is an equilibrium phenomenon — i.e., investors optimally decide to delegate their portfolio choices to the advisor balancing out the costs and benefits of leaving discretion to the expert. This is different from the approach taken in Gennaioli *et al.* (2013) where trust is modeled as an exogenous parameter that reduces the investors' perception of the riskiness of a given investment.

is, they delegate the portfolio decision to the financial advisor, but impose a lower cap on the proportion of their wealth that the advisor can allocate to the risky asset. Overall, the fact that portfolio allocations are perceived as complements hinders the relationship between the advisor and each investor. In other words, the presence of an investor generates a negative externality on the other investor. Investors would be better off (obtain a higher expected utility) under exclusive financial advice.

In contrast, when the financial advisor perceives investors' portfolio choices as substitutes she has an incentive to induce each investor to take *less* risk than in the case of exclusive financial advice. This helps mitigate the conflict of interests between the investors and the financial advisor, but only up to a point. When the degree of substitutability is relatively low, investors trust more the financial advisor and delegate more by increasing the cap on the proportion of their wealth the investor can allocate to the risky asset. In this case, the presence of one investor generates a positive externality on the other investor. Investors are better off when financial advice is non-exclusive than when it is exclusive. However, when the degree of substitutability is sufficiently high, the direction of the misalignment of incentives changes. In this case, the advisor actually prefers that investors invest too little in the risky asset (relative to their optimal allocation) generating a conflict of interest of a different sort. Investors respond by changing the investment rule they propose to the financial advisor. They still delegate the portfolio decision to the financial advisor, but now they impose a floor on the proportion of their wealth that is invested in the risky asset. Whether investors are better off than in the case of exclusive financial advice depends on the degree of substitutability. The higher it is, the larger the misalignment of incentives, the less investor delegate by imposing a higher floor and the lower the investors' expected utility.

As in the case of exclusive financial advice, when investors rely on an common financial advisor, more uncertainty about the state of the world makes investors more willing to delegate to the financial advisor. One possible interpretation of this result is that investors with poor financial literacy rely more often on delegated portfolio management, while more financially educated people are more likely to invest by themselves. These predictions square with the empirical evidence collected by Hackethal et al. (2010) showing that a German bank's less-educated customers were more likely to rely on investment advice, and with that gathered by Calcagno an Monticoni (2013) finding that in Italy more educated people and those working in the financial sector are more likely to invest by themselves.

The agency problem between investors and their money managers has been extensively studied by the earlier literature — see, e.g., Stracca (2005) and Inderst and Ottaviani (2012b) for recent surveys of this literature. Existing models highlight many important aspects of standard delegated portfolio management by studying how investors should optimally design contracts (remuneration schemes) for money managers. Following the moral-hazard tradition some of these models assume that the money manager chooses the riskiness and/or the expected return of his client's portfolio, and that this choice is unobservable to the investor, who then needs to design a second-best contract that motivates the expert to choose the right action (e.g., Adamati and Pfleiderer, 1997; Stoughton 1993; Palomino and Prat, 2003; and Palomino and Uhlig, 2006). In the adverse selection framework, Allen (1985) and Bhattacharya and Pfleiderer (1985) are the first to propose models where a better informed advisor must be solicited to reveal superior information about the rate of return and/or the riskiness of a financial asset to an uninformed investor. These papers show that optimal contracts do not achieve the first-best solution due to the standard trade-off between efficiency and rents (see also Allen and Gorton, 1993; and Das and Sundaram, 1998). The present paper follows the adverse selection approach. However, in contrast with the paper mentioned above, we assume that money transfer between investors and their money managers are not feasible.[4] A similar approach has been taken in the applied cheap talk literature — see, e.g., the survey by Inderst and Ottaviani (2012b) — where investors have no commitment power *vis-à-vis* their financial advisors. We believe that our contribution to this literature is twofold. First, by looking at the mechanism design version of this game, we provide a (second best) benchmark that only focuses on the inefficiencies stemming from the natural asymmetry of information between investors and their advisors, and neglects the additional ones arising from the lack of commitment.[5] Second, we will argue that our commitment assumption is not too strong since our equilibrium outcomes can be implemented by an extremely simple form of delegated portfolio management that is robust to renegotiation as long as investors are required to pay sufficiently large disinvestment fees (see Section 4.3). Finally, and most importantly, while the cheap talk literature has neglected issues of non-exclusivity, this is key to our analysis.

Most of our comparative statics can help design new empirical and experimental tests that may shed light on the relationship between financial advice, non-exclusivity and investors' financial education. In this respect, the paper that is closest in spirit to our analysis is Asparouhova et al. (2013). They also model delegated portfolio management as non-exclusive, but take a general equilibrium approach. In their model managers compete to attract investors by offering bundles of portfolio allocations and intermediation fees, investors can buy at linear prices any combination of portfolios they want. By looking at the general equilibrium implications of non-exclusive advice, they offer a number of interesting predictions (that are then tested through an experiment) on the way competitive money managers should behave both on the pricing and product design sides. However, in their model there is no asymmetric information: delegated portfolio management is not an endogenous result, but rather an assumption. In this sense, our models are complementary.

The paper is structured as follows. Section 2 lays down the baseline model with symmetric investors. Section 3 studies the exclusivity benchmark. In Section 4 we characterize equilibria with non-exclusive financial advice. In Section 5 we provide the comparative statics analysis

[4] The assumption that money transfers between two or more parties to a contract are not possible has been used in the literature on optimal delegation in organizations (e.g., Alonso and Matouschek, 2008; Dessein, 2002; Martimort and Semenov, 2006; and Melumad and Shibano, 1996).

[5] On the issue of commitment versus cheap talk see for instance Goltsman *et al.* (2009).

and compare the regimes with and without exclusive analysis. Section 6 extends the analysis to the case of heterogeneous investors. Section 7 concludes. All proofs are in the Appendix.

2. The model

Players and environment. Consider two identical investors (each denoted by $i = 1, 2$) with initial wealth normalized to 1. There is only one risky investment opportunity (e.g., equities, funds, structured products and so on) and the riskless asset. The stochastic return of the risky asset $\tilde{r}$ is normally distributed with mean $\mu > 0$ and variance σ^2. The riskless asset pays the riskfree rate r_f, with $\mu > r_f \geq 1$.

Due to the lack of proper financial education, investors must rely on a (common) financial advisor to make their investment choices. The advisor is better informed than the investors about the variance σ^2 of the risky investment (the state of the world). More precisely, while the expected return μ is common knowledge, the variance σ^2 cannot be assessed with certainty by the investors who need to rely on the superior knowledge of the expert. In the absence of financial advice, investors have only a symmetric prior about the state of the world: they believe that σ^2 distributes uniformly over the compact support $\Sigma \equiv [1 - \Delta, 1 + \Delta]$, with $\Delta \in (0, 1)$. This prior is common knowledge.

The idea is that non-institutional investors (such as households), with limited access to detailed information about asset returns, are less able to quantify the risk carried by financial activities rather than their expected returns. This assumption is standard in the delegated portfolio management literature. Palomino and Uhlig (2007), for instance, argue that mutual fund regulation does not require funds to disclose their portfolio very often, and managers window dress around disclosure dates. Therefore, for young funds with a short track record, estimating the return volatility may not be possible in the absence of insider information. Moreover, if one considers private equity funds, their return volatility may be difficult to estimate since their net asset value is not very often observed.[6,7]

Contracting. Contracts that require state contingent monetary transfers between the advisor and his clients are not enforceable — i.e., there cannot be fees contingent on the information transmitted by the expert to the investors (fixed fees are normalized to zero without loss of generality). We focus on a simple class of (bilateral) direct mechanisms. Let α_i be the fraction

[6] For example, the European Private Equity and Venture Capital Association (EVCA) advises funds to release net asset values on a quarterly basis — see, e.g., the EVCA Reporting Guidelines (2000). Furthermore, these reported values are often based on funds' self valuation of their portfolio companies — see, e.g., the EVCA Valuation Guidelines (2004).

[7] Alternatively, one can imagine that unsophisticated people find it harder to estimate the return volatility of an asset rather than estimating its expected return. This is because the former estimate relies on the latter. Hence, the probability of making mistakes in calculating the return volatility of an asset cannot be smaller than that of calculating its expected return. We normalize the probability of a wrong estimation of the average return of the risky asset to zero, and capture the error made in estimating the variance with the parameter Δ, which proxies the variance of the uniform prior on Σ.

of wealth that investor i allocates to the risky asset, or his risk exposure. Each investor i chooses a direct mechanism $\mathcal{M}_i \equiv \{\alpha_i(m_i)\}_{m_i \in \Sigma}$, with $\alpha_i(.) : \Sigma \rightarrow \Re$, which specifies a portfolio allocation $\alpha_i(m_i)$ for any (private) report $m_i \in \Sigma$ made by the advisor to investor i about the state of the world σ^2. As standard in this literature, mechanisms are restricted to be piecewise differentiable and continuous. The expert cannot refuse advice to his clients: an intrinsic common agency game.[8]

The use of direct mechanisms allows us to rely on intuitive and easy to characterize incentive constraints. Yet, as we will argue in Section 4.3, the equilibrium outcomes that will be characterized throughout can be implemented by simple (indirect) mechanisms such that each investor announces a choice set within which the advisor can pick his most preferred allocation.

Timing. The timing of the game is as follows:

- $(t = 0)$ Nature draws σ^2 and only the advisor observes its realization.
- $(t = 1)$ Each investor i announces (and commits to) a mechanism $\mathcal{M}_i$. These announcements are simultaneous.
- $(t = 2)$ The advisor (privately) reports m_i to each investors i. Investment choices are made according to the mechanisms chosen at $t = 1$.
- $(t = 3)$ Asset returns materialize.

The commitment assumption is standard in the mechanism design literature that studies delegation in the absence of monetary incentives. In particular, it allows to avoid the typical selection issue of cheap talk games (e.g., Crawford and Sobel, 1982, among others), and is often motivated with a reputation argument. That is, the relationship between an investor and his financial advisor is usually long-lasting (due to switching costs). In addition, we will argue in Section 4.3 that our equilibrium mechanisms are robust to the threat of renegotiation once sufficiently large disinvestment fees are imposed.

Preferences and conflict of interests. Each investor's utility function is CARA — i.e., for any level of wealth w_i

$$u(w_i) = 1 - e^{-\gamma w_i},$$

where $\gamma > 0$ measures the investors' risk attitude. Therefore, for any given state of the world $\sigma^2 \in \Sigma$, the investors' expected utility can be described by first two moments of the asset return

[8] For simplicity, we have neglected the typical issue of common agency games where principals may offer contracts that are contingent on the agent's report not only on the (physical) state of nature, but also on the offers that he has received by the other principals — see, e.g., Attar *et al.* (2011), Martimort and Stole (2002, 2003) and Pavan and Calzolari (2009). A key difference between our model and this literature is that we rule out monetary transfers that typically play a key role in these models. In this sense, extending their approach to our analysis might be a non obvious task, which goes behind the scope of this paper.

— i.e., the mean and the variance of the distribution of wealth. Hence,

$$u(\alpha_i, \sigma^2) = \mathbb{E}\left[\alpha_i \tilde{r} + (1-\alpha_i)\, r_f\right] - \frac{\gamma}{2}\mathbb{E}\left[\alpha_i\left(\tilde{r}-\mu\right)\right]^2 = \alpha_i\left(\mu - r_f\right) + r_f - \frac{\gamma}{2}\alpha_i^2\sigma^2. \tag{2.1}$$

Absent asymmetric information, the investors' optimal asset allocation solves the following maximization problem

$$\max_{\alpha_i \in [0,1]} \left\{\alpha_i\left(\mu - r_f\right) + r_f - \frac{\gamma}{2}\alpha_i^2\sigma^2\right\},$$

which yields the standard mean-variance allocation (i.e., the first-best benchmark)

$$\alpha^F(\sigma^2) = \frac{\mu - r_f}{\gamma\sigma^2}, \tag{2.2}$$

Notice that at $\sigma^2 = 1$, the first-best asset allocation $\alpha^F(1)$ can be interpreted as a measure of the risk premium per unit of risk aversion. To save on notation, throughout we will define this index by $\pi \equiv \alpha^F(1)$.

To compare in the clearest possible way our analysis with earlier models, we assume that the advisor's preferences are represented by the following quadratic loss function that depends symmetrically on the portfolio choice of both investors — i.e.,

$$v\left(\boldsymbol{\alpha}, \sigma^2\right) = -\frac{1}{2}\sum_{i=1,2}\left[\alpha_i - (1+\lambda)\,\alpha^F(\sigma^2)\right]^2 - \theta\alpha_1\alpha_2, \tag{2.3}$$

where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$. The term $(1+\lambda)\,\alpha^F(\sigma^2)$ represents the advisor's ideal investment level on each investor i under exclusive advice.

This loss function has a simple interpretation.[9] Under exclusive advice (i.e., if there is only one client), the parameter $\lambda \in [0, 1)$ measures the advisor's intrinsic bias relative to the investor's ideal portfolio choice — i.e., only when $\lambda = 0$ their interests are perfectly aligned. In this case, the investor's optimal allocation of wealth and the advisor's ideal point coincide: even if the former does not know the state of the world, there is no need to elicit truthful information revelation. This means that for $\lambda = 0$ the optimal mechanism under exclusive advice entails full delegation and it implements the first-best allocation. By contrast, when $\lambda > 0$, there is a misalignment of preferences between the advisor's and the depositors' risk attitude. This conflict has a natural interpretation highlighted in Ottaviani (2000): providers of financial products (e.g., funds) often give sales commissions to the banks selling their products that are increasing on the amount of sales. As a result, one can expect money managers to adjust the risk level in order to maximize their (implicit or explicit) compensation, which gives rise to an agency model in which the agent controls risk which may induce λ to be positive.

However, when financial advice is non-exclusive, the presence of multiple investors dealing with a common advisor may create additional externalities, through the advisor's preference

[9] Similar loss functions are used in the cheap-talk literature (e.g., Crawford and Sobel, 1982).

structure, that we capture with the interaction term $\theta\alpha_1\alpha_2$, where $\theta \in (-1, 1)$. Specifically, when $\theta < 0$, the investors' portfolio choices are perceived as complements by the advisor — i.e.,

$$\frac{\partial^2 v\left(\boldsymbol{\alpha}, \sigma^2\right)}{\partial\alpha_1\partial\alpha_2} > 0 \quad \forall\boldsymbol{\alpha}.$$

In this case, the advisor prefers investor i to allocate a larger fraction of his wealth into the risky activity when investor j does so too. This complementarity may arise either because providers of the financial product pay commissions to the bank that reward not only the achievement of investment targets on each single client, but that also award premia that are increasing with the size of the clientele gathered by the bank — e.g., when funds reward the achievement of clients' targets. Alternatively, it could simply reflect the 'empire building' desire of the advisor — e.g., a large portfolio of active clients may provide him a stronger bargaining position *vis-à-vis* the banks' top management.

By contrast, when $\theta > 0$ the investors' portfolio choices are perceived as substitutes by the advisor — i.e.,

$$\frac{\partial^2 v\left(\boldsymbol{\alpha}, \sigma^2\right)}{\partial\alpha_1\partial\alpha_2} < 0 \quad \forall\boldsymbol{\alpha}.$$

In this case, the advisor would like investor i to allocate a lower fraction of his wealth into the risky activity when investor j's investment into the same activity increases. This may be either due to exogenous liquidity needs — i.e., the advisor may wish to achieve liquidity targets (perhaps mandated by binding regulatory constraints) — or it may just reflect his concerns about reputational losses following unsuitable sales.[10] This may happen, for example, when the bank's top management is sensible to clients' dissatisfaction, especially when complains about 'misselling' may involve litigation costs (perhaps due to class actions) or even the loss of the client.[11]

We make the following assumption on the parameters of the model.

Assumption 1. $1 - \lambda + 2\theta > 0$ and $\pi < 1 - \Delta$.

This assumption simplifies the analysis as it implies that taking short positions on any of the two assets is never optimal — i.e., under the optimal mechanism $0 \leq \alpha_i \leq 1$ for $i = 1, 2$. Finally, we assume that the structure of the advisor's preferences is common knowledge — i.e., both investors know λ and θ. This implies that investors are wary of the conflict of interests

[10] Gennaioli *et al.* (2013), for instance, argue that money mangers are particularly sensible to their reputation. In our model we are implicitly assuming that reputation may be crucially affected by the size of an advisor's clientele via network effects that may arise from information spillovers between clients.

[11] Regulations for consumer financial services that take seriously into account complaints about deceptive advice are widespread. In the United States, the 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act created the Consumer Financial Protection Bureau to write and enforce rules, conduct examinations, and track consumer complaints. In the UK a revised regulatory architecture is expected to replace the Financial Services Authority with a new Financial Conduct Authority empowered to order withdrawal of financial products or misleading promotions (Financial Services Authority).

with the advisor.[12]

The equilibrium concept is Perfect Bayesian Equilibrium.

3. Exclusive financial advice

We begin with the analysis of the case in which the advisor provides financial advice to one investor only. The analysis of this case will help us gain insights on the basic trade-offs that determine the equilibrium portfolio choices. It will also provide a benchmark against which we will compare the results obtained in the case of non-exclusive financial advice.[13]

Since the advisor's risk preferences are misaligned with those of the investor — i.e., $\lambda > 0$ — the expert has an incentive to manipulate his report about the state of the world. To see why, suppose that in an attempt to implement the first-best allocation $\alpha^F(\sigma^2)$, the investor chooses the following mechanism

$$\widehat{\alpha}(m) = \frac{\pi}{m},\ \forall m.$$

The advisor has then an obvious incentive to lie by reporting

$$m = \frac{\sigma^2}{1+\lambda} < \sigma^2,$$

rather than the true variance of the risky asset σ^2. By doing so, the advisor obtains exactly his ideal point. To prevent this behavior, the optimal mechanism must elicit a truthful report — i.e., it must be incentive compatible for the advisor. Observe that given a mechanism $\alpha(.)$, the advisor's utility when he reports m to the investor and the state of the world is σ^2 is given by

$$v(\alpha(m), \sigma^2) \equiv -\frac{1}{2}\left[\alpha(m) - (1+\lambda)\,\alpha^F(\sigma^2)\right]^2.$$

Incentive compatibility requires that

$$-\frac{1}{2}\left[\alpha(\sigma^2) - (1+\lambda)\,\alpha^F(\sigma^2)\right]^2 \geq -\frac{1}{2}\left[\alpha(m) - (1+\lambda)\,\alpha^F(\sigma^2)\right]^2,\ \forall(\sigma^2, m) \in \Sigma^2.$$

Within the class of continuous mechanisms, the above global incentive constraint can be replaced by the following local first-order condition (see, e.g., Martimort and Semenov, 2006)

$$\frac{\partial}{\partial m} v(\alpha(\sigma^2), \sigma^2) = 0 \quad \Leftrightarrow \quad \left[\alpha(\sigma^2) - (1+\lambda)\,\alpha^F(\sigma^2)\right]\dot{\alpha}(\sigma^2) = 0. \qquad (3.1)$$

That is, the asset allocation announced by the investor must be such that the advisor's utility

[12] In reality, however, investors may not be perfectly aware of fine details of their advisors' preference structure. But, if contracts contingent on details that are not directly related to nature of the financial product on sale are not enforceable, a simple interpretation of our model is that investors reason as if they are facing the representative advisor — i.e., for which λ and θ are averages taken from the population of possible advisors' types.

[13] Observe that the equilibrium of the game with a single investor is equivalent to the outcome of the common agency game when the advisor's utility is separable across clients — i.e., $\theta = 0$.

is maximized when he truthfully reports the state of the world. The incentive compatibility condition (3.1) is satisfied by two interesting classes of functions: the pooling ones, where the asset allocation is unresponsive to the state of the world; and the separating ones, which mandate an investment that coincides with the advisor's ideal point — i.e.,

$$\alpha_E^A(\sigma^2) = (1+\lambda)\alpha^F(\sigma^2).$$

An optimal investment might may also combine these two schemes, so that it is sometimes optimal for the investor to let the advisor pick his most preferred asset allocation.

Let $\mathcal{P} = \cup_{k=1}^K \mathcal{P}_k$ be the union of all K subsets of Σ in which the investor pools by choosing α_k for every $\sigma^2 \in \mathcal{P}_k$. The investor's maximization problem is

$$\max_{(\mathcal{P}_k,\alpha_k)_{k=1}^K, K} \left\{ \int_{\Sigma\setminus\mathcal{P}} \alpha_E^A(\sigma^2)\left[\pi - \frac{\alpha_E^A(\sigma^2)\sigma^2}{2}\right] d\sigma^2 + \sum_{k=1}^K \int_{\mathcal{P}_k} \alpha_k \left[\pi - \frac{\alpha_k\sigma^2}{2}\right] d\sigma^2 \right\}.$$

subject to $K \in \mathbb{Z}$, $\mathcal{P}_k \subseteq \Sigma$ and $\alpha_k \in [0,1]$ for every $k = 1, .., K$.[14]

Proposition 1. *The optimal mechanism when financial advice is exclusive, $\mathcal{M}_E^* \equiv (\alpha_E^*(m))_{m\in\Sigma}$, satisfies the following properties. If $\lambda < \Delta$, then the investor partially delegates the investment decision to the financial advisor. Specifically, his investment decision is given by*

$$\alpha_E^*(\sigma^2) = \begin{cases} \alpha_E^* & \text{if} \quad \sigma^2 \le x_E^* \\ \alpha_E^A(\sigma^2) & \text{if} \quad \sigma^2 > x_E^* \end{cases},$$

with $x_E^* = \frac{1+\lambda}{1-\lambda}(1-\Delta) \in (1-\Delta, 1+\Delta)$ and $\alpha_E^* = \pi\frac{1-\lambda}{1-\Delta}$. If $\lambda \ge \Delta$, then the investor totally ignores the information provided by the financial advisor and invests $\alpha_E^*(\sigma^2) = \pi$ for all $\sigma^2 \in \Sigma$.

The optimal portfolio choice under asymmetric information is shaped by two contrasting forces. To induce a truthful report by the advisor, the investor must either force a pooling allocation, or he must allow the expert to get his ideal investment choice. Both these schemes depart from the investor's first best allocation, and are thus costly to him. On the one hand, an investment rule unresponsive to the state of the world — i.e., the pooling one — is costly to the investor because he is risk averse, and thus would like to invest an amount of wealth into the risky asset tailored to its return volatility. On the other hand, the cost of linking the investment strategy to the state of the world — i.e., a separating outcome — stems for the fact that such allocation must coincide with the advisor's ideal investment to guarantee truthful information revelation. But, because preferences are misaligned, this carries more risk than what the investor would like to bear.

The relative magnitude of these costs determines the structure of the optimal asset allocation. Proposition 1 states that, when the conflict of interest between the investor and his advisor is not

[14] Hereafter the symbol $\mathbb{Z}$ will denote the set of all integers.

very strong ($\lambda < \Delta$), it is optimal for the investor to leave discretion to the advisor and enable him to implement his ideal point if the volatility of the risky asset is larger than the threshold x_E^*. This is because the difference between the players' ideal points is less pronounced when the realized variance σ^2 is large. Hence, the cost of delegation is relatively less severe than the cost of pooling to the investor, who prefers a portfolio that covaries with the state of the world. By contrast, when σ^2 is low, the agency conflict is harder to be reconciled with a separating allocation: in these states of the world the advisor's most preferred investment into the risky asset is much larger than what the investor wishes. Thus, it is optimal for the latter to force a flat rule. Notice, however, that when λ is large enough, the objectives of the two players diverge so much that the cost of delegation always outperforms that of basing financial decisions on the prior alone. In this case, the optimal asset allocation rule requires a fully pooling allocation — i.e., $x_E^* = 1 + \Delta$.

A first implication of the model is that, under-exclusivity, uninformed investors rely more often on financial advise when buying assets whose return volatility is large. Another interesting prediction is that, compared to the first-best benchmark, there is under-investment into the risky activity when its return volatility is low, and over-investment otherwise. In addition the pooling region expands (i.e., x_E^* increases) when λ becomes larger because the misalignment of preferences between the investor and the advisor becomes more severe. The impact of the risk premium and the risk aversion coefficient on the optimal portfolio allocation are as in the standard mean-variance analysis. What is perhaps less obvious is the impact of λ on the optimal portfolio allocation. Clearly, a larger intrinsic bias λ increases the optimal (risky) investment within the delegation region. But, the opposite holds true in the pooling region. In this case, a larger λ reduces the pooling allocation because it exacerbates the conflict of interest between the advisor and the investor. This leads the latter to adopt a more conservative investment strategy in order to soften his exposure to the former's opportunistic behavior.

Finally, it can be easily seen that a larger Δ induces delegation in a larger set of circumstances[15]: less transparency or poor financial literacy of investors induce more reliance on experts and money managers. Noteworthy, the investment into the risky asset in the pooling region becomes larger when uncertainty increases.

4. Non-exclusive financial advice

Consider now the case where the advisor deals simultaneously with two identical investors. To characterize the equilibria of this game, we first study how non-exclusivity of the financial advice modifies the advisor's incentive compatibility constraint and his ideal investment choices.

Given mechanisms $\alpha_1(.)$ and $\alpha_2(.)$, the advisor' utility if he reports m_1 and m_2 to investors

[15] Indeed, the area where the investor forces a pooling allocation, which is measured by the ratio $\frac{x_E^* - (1-\Delta)}{2\Delta} = \frac{1-\Delta}{\Delta} \frac{\lambda}{1-\lambda}$, shrinks when Δ increases.

when the true variance is σ^2 is given by

$$v(\alpha_1(m_1), \alpha_2(m_2), \sigma^2) = -\frac{1}{2} \sum_{i=1,2} \left[\alpha_i(m_i) - (1+\lambda)\, \alpha^F(\sigma^2)\right]^2 - \theta \alpha_1(m_1) \alpha_2(m_2).$$

Using the same logic as before, the (local) incentive compatibility conditions, necessary and sufficient to guarantee that the advisor truthfully reports σ^2 to each investor i are

$$\frac{\partial}{\partial m_i} v(\alpha_1(\sigma^2), \alpha_2(\sigma^2), \sigma^2) = 0 \quad \Leftrightarrow \quad \left[\alpha_i(\sigma^2) - (1+\lambda)\, \alpha^F(\sigma^2) + \theta \alpha_j(\sigma^2)\right] \dot{\alpha}_i(\sigma^2) = 0$$

for $i = 1, 2$. Hence, for any given mechanism $\alpha_j(\sigma^2)$ chosen by investor j, an incentive compatible mechanism for investor i is either flat — i.e., $\dot{\alpha}_i(\sigma^2) = 0$ — or it requires

$$\alpha_i(\sigma^2) = \alpha_E^A(\sigma^2) - \theta \alpha_j(\sigma^2) \equiv \alpha_i^A(\sigma^2), \tag{4.1}$$

which is the advisor's ideal point on investor i's portfolio choice (given the choice of investor j).

Equation (4.1) suggests an important difference between the case in which financial advice is exclusive and the case where financial advice is non-exclusive. When the advisor serves both investors, his ideal point on investor i is equal to the individual (or intrinsic) target $\alpha_E^A(\sigma^2)$ net of the externality $\theta \alpha_j(\sigma^2)$. Hence, the expert's incentive to understate or overstate the value of σ^2 depends both on the sign and the magnitude of the interaction term $\theta \alpha_j(\sigma^2)$. As we will see below, both crucially affect the shape of the mechanisms investors choose in equilibrium as well as their portfolio allocations. Notice that if $\dot{\alpha}_i(\sigma^2) \neq 0$ for each $i = 1, 2$, then it must be the case that both investors choose the same asset allocation

$$\alpha_N^A(\sigma^2) = \frac{\alpha_E^A(\sigma^2)}{1+\theta}, \tag{4.2}$$

which is (strictly) decreasing in σ^2. Essentially, if both investors fully delegate their portfolio choices to the common advisor, they invest the same fraction of wealth into the risky asset. This is because the advisor's preferences are symmetric with respect to investment choices, and investors feature the same attitude towards risk — i.e., they have an identical first-best allocation.

Since investors are identical we look for symmetric equilibria where both investors choose the same mechanism $\mathcal{M}_N^*$, which requires pooling in the subset $\mathcal{P}_N^* \subseteq \Sigma$ and, within this subset, allocates a share α_N^* of wealth to the risky asset. The mechanism $\mathcal{M}_N^*$ must solve the maximization problem of each investor given that the other investor also chooses $\mathcal{M}_N^*$. Let $\mathcal{P}_i = \cup_{k=1}^{K_i} \mathcal{P}_{i,k}$ be the union of all K_i subsets of Σ in which investor i pools by choosing $\alpha_{i,k}$ for every $\sigma^2 \in \mathcal{P}_{i,k}$. Investor i's maximization problem is

$$\max_{(\mathcal{P}_{k,i},\alpha_{k,i})_{k=1}^{K_i},K_i} \left\{ \int_{(\Sigma\backslash\mathcal{P}_i)\cap\mathcal{P}_N^*} \alpha_i^A(\sigma^2)\left[\pi - \frac{\alpha_i^A(\sigma^2)\sigma^2}{2}\right] d\sigma^2 + \right.$$

$$\left. + \int_{(\Sigma\backslash\mathcal{P}_i)\cap(\Sigma\backslash\mathcal{P}_N^*)} \alpha_N^A(\sigma^2)\left[\pi - \frac{\alpha_N^A(\sigma^2)\sigma^2}{2}\right] d\sigma^2 + \sum_{k=1}^{K_i} \int_{\mathcal{P}_{i,k}} \alpha_{i,k}\left[\pi - \frac{\alpha_{i,k}\sigma^2}{2}\right] d\sigma^2 \right\}.$$

subject to $K_i \in \mathbb{Z}$, $\mathcal{P}_{k,i} \subseteq \Sigma$ and $\alpha_{i,k} \in [0,1]$ $\forall k = 1,..,K_i$.

We start the characterization of the equilibrium with the following lemma.

Lemma 1. *In any symmetric equilibrium, neither the pooling region nor the delegation region of the mechanism chosen by the investors are strictly contained in* Σ.

Lemma 1 implies that in a symmetric equilibrium only two types of equilibria with partial delegation may arise: an equilibrium where each investor pools for low values of σ^2, and delegates otherwise (exactly as in the exclusivity benchmark), and an equilibrium where each investor delegates for low values of σ^2, and pools otherwise. Of course, in addition to these outcomes there can also exist symmetric equilibria with full pooling or with full delegation.

We next consider separately the case in which portfolio choices are perceived as complements by the advisor and the case where they are perceived as substitutes.

4.1. Portfolio choices perceived as complements by the financial advisor

The case where portfolio choices are perceived as complements by the advisor corresponds in our model to the case where $\theta < 0$. This is because when $\theta < 0$ an increase in α_j expands the advisor's ideal portfolio choice of investor i: the more risk one investor takes, the higher the incentive of the advisor to induce the other investor to also take more risk. This can be easily seen, for example, by direct inspection of $\alpha_i^A(\sigma^2)$ provided in (4.1).

Because of this effect, the common advisor has an incentive to induce each investor to take *even more* risk than in the case of exclusive financial advice. Observe that $\alpha_i^A(\sigma^2) \geq \alpha_E^A(\sigma^2)$ and $\alpha_N^A(\sigma^2) > \alpha_E^A(\sigma^2)$ when $\theta < 0$. Hence, in this case, the incentives of the financial advisor and those of the individual investors are even less aligned than under exclusive financial advice. Moreover, the misalignment of incentives exacerbates as θ decreases. This suggests that under non-exclusive advice investors should trust less the advisor and be less keen to delegate than under exclusive advice. It also suggests investors delegate less as θ decreases. The next proposition shows this is precisely what investors do when choosing the mechanism that will govern their relationship with the advisor.

Proposition 2. *Suppose that* $\theta < 0$. *The game with non-exclusive financial advice has a unique symmetric equilibrium. The mechanism chosen by investors in that equilibrium has the following* *properties. If $\theta > (\lambda - \Delta)/(1+\Delta)$, investors partially delegate their investment decisions to the financial advisor and each invests*

$$\alpha_N^*(\sigma^2) = \begin{cases} \alpha_N^* & \text{if} \quad \sigma^2 \leq x_N^* \\ \alpha_N^A(\sigma^2) & \text{if} \quad \sigma^2 > x_N^* \end{cases},$$

with $x_N^* \equiv \frac{1+\lambda}{1-\lambda+2\theta}(1-\Delta) \in (1-\Delta, 1+\Delta)$ and $\alpha_N^* = \frac{\pi}{1-\Delta}\frac{1-\lambda+2\theta}{1+\theta}$. If $\theta \leq (\lambda-\Delta)/(1+\Delta)$, investors ignore the advisor's reports and choose $\alpha_N^*(\sigma^2) = \pi$ for all σ^2.

When $\lambda > \Delta$, conditions $\theta < 0$ and $\theta > (\lambda - \Delta)/(1+\Delta)$ cannot be satisfied simultaneously. As in the case of exclusive financial advice, when $\lambda > \Delta$ investors never delegate the investment decision to the advisor. In this case, the conflict of interest that stems from the advisor's intrinsic bias towards an excessive risk exposure is already so strong that delegation is never optimal. Hence, full pooling emerges at equilibrium.

However, when $\lambda < \Delta$ delegation emerges in equilibrium as long as the complementarity of portfolio choices is not to strong (i.e., as long as θ is not too low). Nevertheless, it is a more limited form of delegation than that in the case of exclusive advice, since $x_N^* > x_E^*$. Indeed, as argued above, in this region of parameters the expert's ideal point on each investor is larger with non-exclusive advice than with exclusive advice. Hence, the common advisor has an additional reason to understate the true variance, which makes the investors less keen to trust him and to delegate.

Moreover, as expected, investors delegate less as θ decreases. To see why, observe that the condition $\theta > (\lambda - \Delta)/(1+\Delta)$ is less likely to be satisfied for lower values of θ, and that the threshold x_N^* decreases with θ. In words, a lower θ makes the advisor more willing to induce excessive risk taking by both investors, which exacerbates the conflict of interest between them and calls for less delegation.

Finally, as in the exclusivity benchmark, delegation becomes more likely as Δ increases. A larger Δ, which reflects more uncertainty about the state of the world, amplifies the informative advantage of the advisor and thus obliges the investor to rely more often on him.[16]

4.2. Portfolio choices perceived as substitutes

Consider now the case where portfolio choices are perceived as substitutes by the financial advisor — i.e., the case where $\theta > 0$. In this region of parameters, an increase in α_j reduces the advisor's ideal portfolio choice of investor i: the more risk one investor takes, the *lower* the incentive of the advisor to induce the other investor to take more risk. Once again, this can be easily seen by direct inspection of $\alpha_i^A(\sigma^2)$. Observe that the direction of the externality created by one investor on the other is opposite to that when portfolio choices are complements.

A consequence of this effect is that in the case where portfolios are perceived as substitutes, the common advisor has an incentive to induce each investor to take *less* risk than in the case

[16]Indeed, the ratio $\frac{x_N^*-(1-\Delta)}{2\Delta} = \frac{1-\Delta}{\Delta}\frac{\lambda-\theta}{1-\lambda+2\theta}$ is decreasing in Δ.

of exclusive financial advice. Observe that $\alpha_i^A(\sigma^2) \leq \alpha_E^A(\sigma^2)$ and $\alpha_N^A(\sigma^2) < \alpha_E^A(\sigma^2)$ when $\theta > 0$. Hence, in this case, non-exclusivity of financial advice mitigates the conflict of interest between the advisor and investors. This suggest that investors should trust more the advisor and delegate more. The following proposition shows that this is true, but only up to a point. If θ is too high, the advisor may actually prefer that investors invest too little in the risky asset generating a conflict of interest of a different sort.

Proposition 3. *Suppose $\theta > 0$. The game with non-exclusive financial advice has a unique symmetric equilibrium. The mechanism chosen by investors in that equilibrium has the following properties. If $(\lambda - \Delta)/(1+\Delta) < \theta \leq \lambda$, investors partially delegate their investment decisions to the financial advisor and each invests*

$$\alpha_N^*(\sigma^2) = \begin{cases} \alpha_N^* & \text{if} \quad \sigma^2 \leq x_N^* \\ \alpha_N^A(\sigma^2) & \text{if} \quad \sigma^2 > x_N^* \end{cases},$$

with $x_N^* \equiv \frac{1+\lambda}{1-\lambda+2\theta}(1-\Delta) \in (1-\Delta, 1+\Delta)$ and $\alpha_N^* = \frac{\pi}{1-\Delta}\frac{1-\lambda+2\theta}{1+\theta}$. If $\lambda < \theta < (\lambda+\Delta)/(1-\Delta)$ investors also partially delegate their investment decisions to the financial advisor but each invests

$$\alpha_N^*(\sigma^2) = \begin{cases} \alpha_N^A(\sigma^2) & \text{if} \quad \sigma^2 \leq x_N^* \\ \alpha_N^* & \text{if} \quad \sigma^2 > x_N^* \end{cases},$$

with $x_N^* = \frac{1+\lambda}{1-\lambda+2\theta}(1+\Delta) \in (1-\Delta, 1+\Delta)$ and $\alpha_N^* = \frac{\pi}{1+\Delta}\frac{1-\lambda+2\theta}{1+\theta}$. If $\theta \leq (\lambda - \Delta)/(1+\Delta)$ or $\theta \geq (\lambda+\Delta)/(1-\Delta)$ then investors ignore the advisor's reports and choose $\alpha_N^*(\sigma^2) = \pi$ for all σ^2.

Consider first the case where $\theta < \lambda$ — i.e., the advisor perceives the portfolio allocation of his clients as *weak substitutes*. As expected, delegation is more likely to occur than in the case of exclusive financial advice. Under exclusive financial advice delegation is optimal only if $\lambda < \Delta$, while here it occurs in equilibrium if $(\lambda - \Delta)/(1+\Delta) < \theta$. Moreover, observe that in this case of weak substitutes, investors delegate their portfolio allocation decisions more as θ increases. Indeed, as θ increases, either delegation becomes more likely to occur in equilibrium or when it occurs it is of less constrained form (x_N^* is decreases with θ). As hinted above, this is because a larger θ makes the advisor less willing to induce excessive risk taking by both investors, which mitigates the conflict of interest between them and calls for more delegation. Clearly, if more discretion is left to the advisor as a result of a lower conflict of interest, each client is also more eager to invest more into the risky activity because there is less fear of opportunistic behavior by the advisor.

For $\theta = \lambda$ the unique equilibrium of the game with non-exclusive advice trivially yields the first-best outcome (full delegation). Essentially, in this cutting edge case, the negative externality between the portfolio choices stemming from non-exclusive advice exactly compensates the advisor's incentive to induce excessive risk taking, which stems from his intrinsic bias towards an excessive risk exposure.

Consider now the case where $\theta > \lambda$ — i.e., the advisor perceives the portfolio allocation of his clients as *strong substitutes*. If θ is not too large, partial delegation may emerge again in equilibrium. However, it is a different form of delegation. Observe that in this case, the investor sets a minimum on the fraction of his wealth invested into the risky asset. This contrasts with the form of delegation when portfolio choices are weak substitutes, where investors put a cap on the amount invested into the risky asset. The reason is that, with strong substitutability between investment choices, the ideal point of the advisor on each client falls below the first-best level. Hence, the expert has an incentive to report a variance larger than the true one, so as to induce both clients to take less risk than what they would like to bear. To prevent this type of behavior, in a symmetric equilibrium, it is optimal for both investors to impose a floor on the investment into the risky activity so as to discourage the advisor from over-reporting risk. This leads to a novel type of equilibrium where both investors leave discretion to the advisor when the return volatility of the risky asset is low, and pool when it is sufficiently large.

Finally, as in the exclusivity benchmark and the case where portfolio choices are complements, delegation becomes more likely as Δ increases. However, in contrast to the case where portfolio choices are complements or weak substitutes, when they are strong substitutes the amount of wealth invested into the risk asset within the pooling region α_N^* is decreasing in Δ: a larger uncertainty about the state of the world makes investors less willing to risk.

4.3. A remark on delegated portfolio management

So far, we have characterized the equilibrium outcomes of the game by using *direct* mechanisms. But, are there simpler indirect mechanisms that sustain the portfolio allocations characterized throughout the analysis? Are these indirect mechanisms consistent with real life practices? It turns out that these indirect mechanisms take an extremely simple form: they require investors simply to let the advisor choose an asset allocation within a given (compact) choice set.

Corollary 1. *The indirect mechanisms that implement the equilibrium investment choices characterize in Propositions 1, 2 and 3 have the following features: each investor simply tells the advisor not to invest a fraction of wealth larger than $\overline{\alpha}$ and lower than $\underline{\alpha}$ into the risky activity.*

This result highlights an important feature of our model: investors do not need to play the communication game analyzed above but, to implement the same outcome, they can simply specify the range of investment choices that they are willing to accept, and leave to the advisor full control over the actual portfolio composition — i.e., they choose a simple form of delegated portfolio management.

Most importantly, Corollary 1 has an important implication on the value of commitment in our model. Specifically, if investors delegate their portfolio choices in the way just described above, a simple way to solve the commitment problem would amount to impose sufficiently large disinvestment fees. In the absence of commitment this would indeed prevent an investor from first delegating his asset allocation choice to the expert, observe his actual investment choice thus learning the realization of the variance of the risky asset, and then renege his initial choice by forcing the first best allocation.

5. The investor's expected utility and investment in the risky asset

A natural question that emerges from the above analysis is whether, and to what extent, non-exclusive advice improves the investors' expected utility (welfare) relative to the exclusivity benchmark. In other words, do investors prefer to be in an exclusive relationship with their financial advisors?

It turns out that, in our model, the answer to this question depends on the relative magnitude of the advisor's bias with and without exclusivity. Intuitively, investors are better off when dealing with a common advisor (rather than being in an exclusive relationship with him) if and only if their conflict of interests is exacerbated by exclusive deals. Notice that

$$\left|\alpha_E^A(\sigma^2) - \alpha^F(\sigma^2)\right| > \left|\alpha_N^A(\sigma^2) - \alpha^F(\sigma^2)\right| \quad \Leftrightarrow \quad 2\lambda - \theta\,(1-\lambda) > 0.$$

We can state the following.

Proposition 4. *Investors' expected utility is higher with exclusive financial advice than with non-exclusive financial advice when $\theta < 0$ or $\theta \geq \frac{2\lambda}{1-\lambda} > 0$. Moreover, the investors' expected utility under non-exclusive financial advice increases with θ when $\theta < \lambda$ and decreases with θ when $\theta > \lambda$.*

When investment choices are perceived as complements by the advisor, clients exert a negative externality one on the other: with non-exclusive advice, the expert has an extra reason to understate the state of the world and induce excessive risk taking. As a result, the investors' (ex-ante) utility is higher when they deal with an exclusive advisor. Differently, when investment choices are perceived as substitutes by the advisor, clients might exert a positive externality on each other depending on how strong this substitutability is. More precisely, for moderate substitutability where θ is positive but not too large — i.e., $0 < \theta \leq 2\lambda/(1-\lambda)$ — the advisor's global incentive to induce excessive risk taking is mitigated under non-exclusive advice via the externality channel. By contrast, with strong substitutability — i.e., $\theta > 2\lambda/(1-\lambda)$ — the equilibrium outcome with non-exclusive advice leads investors to underinvest too much into the risky activity, whereby making them better off under exclusive advice: hence investors exert a negative externality one on the other.

Another natural question that emerges from our analysis is whether exclusive financial advice leads investors to invest more (on average) in the risky asset. We can also analyze how the degree of substitutability θ affects such investment under non-exclusive financial advice. In order to provide clear cut empirical implications, we will focus on the impact of investors' uncertainty about the return volatility of their investment (as measured by changes in the parameter Δ) on the expected investment into the risk activity. To this purpose, let

$$\hat{\alpha}_N^* \equiv \int_{1-\Delta}^{1+\Delta} \alpha_N^*(\sigma^2) \frac{d\sigma^2}{2\Delta},$$

denote the average investment with non-exclusivity, and

$$\hat{\alpha}_E^* \equiv \int_{1-\Delta}^{1+\Delta} \alpha_E^* \left(\sigma^2\right) \frac{d\sigma^2}{2\Delta},$$

the average investment in the exclusivity benchmark:

Proposition 5. *There exists $\Delta_1 \in (0,1)$ such that: for $\theta \leq 0$, then $\hat{\alpha}_N^* \geq \hat{\alpha}_E^*$ if and only if $\Delta \geq \Delta_1$; and (ii) for $0 \leq \theta < \lambda$, then $\hat{\alpha}_N^* \geq \hat{\alpha}_E^*$ if and only if $\Delta \leq \Delta_1$. For for $\theta \geq \lambda$, there exists Δ_2 such that $\hat{\alpha}_N^* \geq \hat{\alpha}_E^*$ if and only if $\Delta \leq \Delta_2$. Moreover, $\Delta_2 = 0$ if both λ and θ are not too large.*

This result shows that, ceteris paribus, the impact of non-exclusive advice on the (average) investment into the risky activity depends on the investors' uncertainty about the state of nature. The economic intuition is as follows. Consider first the parameter region where Proposition 2 applies. Recall that as Δ increases (i.e., investors become more uncertain about the state of nature) there is more reliance on the expert. When $\theta > 0$ the common expert is less biased towards an overly risky portfolio than an exclusive advisor. As a consequence, in the non-exclusivity regime less knowledgeable investors (Δ large) allocate a larger share of their wealth into the risky activity than under exclusivity. Differently, more knowledgeable investors (Δ small), which rely more often on their priors and less often on the expert report about the state of nature, under-invest into the risky activity. When $\theta < 0$ the common expert is more biased towards an overly risky asset allocation than an exclusive advisor. As a consequence, in the non-exclusivity regime less knowledgeable investors allocate a larger share of their wealth into the risky activity than under exclusivity, and vice versa.

Next, consider the parameter region where Proposition 3 applies. Here, due to a strong substitutability between portfolio choices, the common expert would like both investors to allocate a lower fraction of their wealth into the risky activity. Recall that also in this region of parameters as Δ increases there is more reliance on the expert. Hence, less knowledgeable investors (Δ large) invest more into the risky activity under exclusivity than when dealing with the common expert. Differently, more knowledgeable investors (Δ small), who rely less often on the common expert invest less into the risky activity. When the difference between λ and θ is large enough, the bias of the common expert in favor of safer investments is so strong that people always invest less into the risky activity under exclusivity than when dealing with the common expert (i.e., regardless of their priors).

6. Heterogeneous investors

So far, we assumed that investors are symmetric. In this section we relax this hypothesis by assuming that they are heterogenous. Specifically, we study two simple examples that bring out a few basic insights on how differences in their priors and risk aversion affect the equilibrium outcome of the game.

6.1. Asymmetric priors

Consider first the case where investors have different priors about the volatility of the risky asset. For simplicity, assume that one investor (say investor 1) is perfectly informed about the realization of the state of the world and thus chooses the first best allocation $\alpha^F(\sigma^2)$. Investor 2, instead, is uncertain about the realization of σ^2 and (as before) has a uniform prior distributed over the support Σ. Hence, a higher (resp. lower) Δ can be interpreted as a larger (resp. smaller) heterogeneity between investors. To focus only on the effect of differential information, we keep assuming that both investors have the same risk aversion coefficient $\gamma > 0$.

Since investor 1 is perfectly informed about the state of the world, only investor 2 has to elicit truthful information revelation from the advisor. Incentive compatibility requires

$$\left[\alpha_2(\sigma^2) - (1+\lambda-\theta)\,\alpha^F(\sigma^2)\right]\dot{\alpha}_2(\sigma^2) = 0,$$

which, as before, is satisfied if $\alpha_2(\sigma^2)$ is flat or if it is equal to the advisor's ideal point

$$\alpha_2^A(\sigma^2) = (1+\lambda-\theta)\,\alpha^F(\sigma^2).$$

To gain insights about the optimal investment choice of investor 2, notice that the difference between the advisor's ideal point and the first-best rule depends only on the sign of $\lambda-\theta$ — i.e.,

$$\alpha_2^A(\sigma^2) - \alpha^F(\sigma^2) = (\lambda-\theta)\,\alpha^F(\sigma^2) \geq 0 \quad \Leftrightarrow \quad \lambda \geq \theta. \tag{6.1}$$

Condition (6.1) suggests that the forces at play in this simple asymmetric environment are similar to those described in the case of symmetric investors. Specifically, when the advisor perceives the investment choices of his clients as complements ($\theta < 0$) or as weak substitutes ($0 < \theta < \lambda$), he has an incentive to induce the uninformed investor to take excessive risk. Hence, the portion of wealth that investor 2 allocates to the risky activity needs to be capped. By contrast, when investment choices are perceived as strong substitutes ($\lambda < \theta$), the advisor has an incentive to induce the uninformed client to underinvest into the risky asset, which in turn requires the portion of wealth allocated to the risky activity to be floored. Clearly, for $\lambda = \theta$ both investors manage to obtain the first-best allocation.

Proposition 6. *When investor* 1 *is fully informed about the state of nature, investor* 2*'s optimal* *portfolio allocation satisfies the following properties. If $\theta \in (\lambda - \Delta, \lambda]$, then*

$$\alpha_2^*(\sigma^2) = \begin{cases} \frac{\pi(1-\lambda+\theta)}{1-\Delta} & \Leftrightarrow \quad \sigma^2 \leq x_2^* \\ \frac{\pi(1+\lambda-\theta)}{\sigma^2} & \Leftrightarrow \quad \sigma^2 > x_2^* \end{cases},$$

with $x_2^* = \frac{(1-\Delta)(1+\lambda-\theta)}{1-\lambda+\theta} \in [1-\Delta, 1+\Delta)$. If $\theta \in (\lambda, \lambda+\Delta)$, then

$$\alpha_2^*(\sigma^2) = \begin{cases} \frac{\pi(1+\lambda-\theta)}{\sigma^2} & \Leftrightarrow \quad \sigma^2 \leq x_2^* \\ \frac{\pi(1-\lambda+\theta)}{1+\Delta} & \Leftrightarrow \quad \sigma^2 > x_2^* \end{cases},$$

with $x_2^* = \frac{(1+\Delta)(1+\lambda-\theta)}{(1-\lambda+\theta)} \in (1-\Delta, 1+\Delta)$. Otherwise $\alpha_2^*(\sigma^2) = \pi$ for all σ^2.

Hence, regardless of the sign of $\lambda - \theta$, the region of parameters where investor 2 delegates to the expert his financial decisions tend to shrink as information becomes more symmetric — i.e., when Δ becomes smaller. In other words, the less informed investor 2 is, the more he needs to rely on the advisor to make his investment choices. The reason is that, relative to the symmetric scenario, in this case the advisor has more incentive to induce the less informed client to take excessive risk, which amplifies the conflict of interest between them, whereby inducing more reliance on external advice.

6.2. Asymmetric risk attitude

We now study an environment where both investors have the same (uniform) prior about the volatility of the risky asset, but they have different attitudes towards risk. For simplicity, consider the case where investor 1 is risk neutral, so that his wealth is entirely invested into the risky asset regardless of the advisor's report m_1, while investor 2's risk averse coefficient is $\gamma > 0$. Hence, a higher γ (resp. lower) captures a larger (resp. smaller) asymmetry between the investors.

Since investor 1 is risk neutral, only investor 2 has to elicit a truthful report from the advisor. Using the same techniques developed in Section 4, it is easy to verify that incentive compatibility requires

$$\left[\alpha_2(\sigma^2) - (1+\lambda)\,\alpha^F(\sigma^2) - \theta\right]\dot{\alpha}_2(\sigma^2) = 0 \quad \forall \sigma^2, \tag{6.2}$$

which is satisfied if either $\alpha_2(\sigma^2)$ is unresponsive to σ^2 or if it is equal to the advisor's ideal point

$$\alpha_2^A(\sigma^2) = (1+\lambda)\,\alpha^F(\sigma^2) - \theta.$$

In contrast to the results stated in the previous sections, with different attitudes towards risk new interesting outcomes may arise. To see why, it is useful to compare the advisor's ideal point with the first-best rule — i.e.,

$$\alpha_2^A(\sigma^2) = (1+\lambda)\,\alpha^F(\sigma^2) - \theta \geq \alpha^F(\sigma^2) \quad \Leftrightarrow \quad \frac{\pi}{\sigma^2} \geq \frac{\theta}{\lambda}. \tag{6.3}$$

Hence, for $\theta \leq 0$ or $\theta > 0$ but not too large, the advisor's ideal point always exceeds the first best choice. This suggests that when the advisor perceives the clients' investment allocation choices as complements — i.e., $\theta \leq 0$ — or as weak substitutes — i.e., $\theta < \lambda\alpha^F(\sigma^2)$ for every σ^2 — investor 2's optimal strategy still requires to cap the portion of wealth invested into the risk asset. When, instead, asset allocations are perceived as very strong substitutes by the advisor — i.e., $\theta > \lambda\alpha^F(\sigma^2)$ for every σ^2 — the advisor's ideal point falls short of the first-best choice. In this case, investor 2's optimal asset allocation requires a floor on the amount invested into the risky asset. However, with heterogenous attitude towards risk, a novel interesting outcome emerges. This case occurs when there exists a $x^* \in (1-\Delta, 1+\Delta)$ such that $\lambda\alpha^F(x^*) = \theta$ — i.e.,

$$1+\Delta > x^* \equiv \frac{\theta}{\lambda\pi} > 1-\Delta,$$

At $\sigma^2 = x^*$ the investor's ideal point coincides with the first best. Hence, $\alpha_2^A(\sigma^2) > \alpha^F(\sigma^2)$ for values of σ^2 lower than x^*: in this region of parameters, investor 2 would like to cap the portion of wealth that he invests into the risky asset. By contrast, $\alpha_2^A(\sigma^2) < \alpha^F(\sigma^2)$ for values of σ^2 that exceed x^*: in this region of parameters investor 2's optimal investment choice is to impose a floor on the amount of wealth that he can invest into the risky asset.

While the first two types of behavior have already been discussed in the case of symmetric investors and heterogenous beliefs, the third one is new and hinges only on the hypothesis that investors are heterogeneous with respect to their risk aversion. This case is interesting because, as we will show in the next proposition, investor 2's optimal asset allocation requires the amount of wealth invested into the risky activity to be capped and floored at the same time.

Proposition 7. *When investor* 1 *is risk neutral, the optimal investment rule chosen by investor* 2 *has the following features. There exist two thresholds* $\overline{\gamma}$ *and* $\underline{\gamma}$*, with* $\overline{\gamma} > \underline{\gamma} > 0$*, such that for every* $\gamma \in (\underline{\gamma}, \overline{\gamma})$*, then*

$$\alpha_2^*(\sigma^2) = \begin{cases} \overline{\alpha}^* & \Leftrightarrow \quad \sigma^2 < \underline{x}^* \\ \frac{(1+\lambda)\pi}{\sigma^2} - \theta & \Leftrightarrow \quad \sigma^2 \in [\underline{x}^*, \overline{x}^*] \\ \underline{\alpha}^* & \Leftrightarrow \quad \sigma^2 > \overline{x}^* \end{cases},$$

with $1+\Delta > \overline{x}^* > \underline{x}^* > 1-\Delta$ *and*

$$\frac{(1+\lambda)\,\pi}{\underline{x}^*} = \theta + \frac{\pi}{\mathbb{E}[\sigma^2|\sigma^2 \leq \underline{x}^*]}, \qquad \frac{(1+\lambda)\,\pi}{\overline{x}^*} = \theta + \frac{\pi}{\mathbb{E}[\sigma^2|\sigma^2 \geq \overline{x}^*]}.$$

$$\underline{\alpha}_2^* = \frac{\pi}{\mathbb{E}[\sigma^2|\sigma^2 \leq \underline{x}^*]}, \qquad \overline{\alpha}_2^P = \frac{\pi}{\mathbb{E}[\sigma^2|\sigma^2 \geq \overline{x}^*]}.$$

Moreover, $\overline{x}^* = 1+\Delta$ *for* $\gamma \leq \underline{\gamma}$ *and* $\underline{x}_2^* = 1-\Delta$ *for* $\gamma \geq \overline{\gamma}$*. Of course,* $\alpha_2^*(\sigma^2) \to 1$ *as* $\gamma \to 0$*.*

Hence, when investors feature small differences in risk aversion (i.e., for γ small enough) they both invest their entire wealth into the risky asset. When differences in risk aversion grow larger, three possible outcomes may occur. For relatively low values of γ the advisor ideal point is still so large that investor 2's optimal asset allocation requires a cap. For intermediate levels of γ, the advisor's ideal point can be larger than the first-best when the return volatility of the risky asset is low. In this case, investor 2 has to impose both a cap and floor to his investment into the risky action. Finally, for large enough differences in risk aversion, the advisor's ideal point falls short of the first-best, whereby inducing investor 2 to impose only a floor on the optimal investment into the risky activity. This suggests that, with heterogeneous risk attitudes: less risk averse people rely less often on financial advice as long as the advisor has an incentive to under-report the return volatility of the risky asset; equilibrium investment strategies may entail delegation only for intermediate values of uncertainty — i.e., more risk averse investors tend to rely less on external advice when buying both relatively riskier and safer assets.

7. Concluding remarks

When people deal with a common financial advisor there may exist externalities between investors, arising from the expert's preferences, that affect their asset allocation choices in a non-obvious way. To highlight this point, we present a simple model of non-exclusive financial advice where two investors rely on a common expert to make their portfolio choices. The expert has better information about the riskiness of the assets in which investors can invest. We characterize the equilibrium of the game when investors commit to continuous asset allocation rules and compare this outcome with that arising under exclusive advice. The model predicts that, when the common expert has preferences that are not separable with respect to their clients' portfolio choices, the equilibria outcome of the game with and without exclusive advice may be substantially different. In contrast to the case of exclusive advice, where the client is more likely to trust the expert when buying assets that are particularly volatile, with non-exclusive advice people may delegate to experts their investment choices when buying assets that are not particularly volatile, and rely upon their imperfect priors otherwise. This discrepancy has novel implications on welfare, investment behavior and the link between financial literacy and investors' propensity to trust experts.

A. Appendix

Proof of Proposition 1. We begin by showing that if the optimal mechanism $\mathcal{M}_E^*$ is such that the investor pools in a subset $\mathcal{P}$ of Σ, then $\mathcal{P}$ cannot be strictly contained in Σ.

Suppose that the investor pools only in the subset $\mathcal{P} \equiv [x, y] \subset \Sigma$, with $1-\Delta < x < y < 1+\Delta$ — i.e., he invests a share α of his wealth into the risky asset for every $\sigma^2 \in \mathcal{P}$. Then, incentive compatibility requires an investment $\alpha_E^A(\sigma^2)$ for every $\sigma^2 \in \Sigma \backslash \mathcal{P}$. The investor's maximization problem is

$$\max_{\mathcal{P},\alpha} \mathcal{W}(\mathcal{P}, \alpha) \equiv \max_{x,y,\alpha} \left\{ \int_x^y \alpha \left[\pi - \frac{\alpha\sigma^2}{2}\right] d\sigma^2 + \int_{\Sigma\backslash[x,y]} \alpha_E^A(\sigma^2) \left[\pi - \frac{\alpha_E^A(\sigma^2)\sigma^2}{2}\right] d\sigma^2 \right\}.$$

subject to $\mathcal{P} \subset \Sigma$ and $\alpha \in [0, 1]$.

The first-order condition with respect to α is

$$\alpha = \frac{\pi}{\mathbb{E}\left[\sigma^2 | \sigma^2 \in \mathcal{P}\right]} \equiv \frac{2\pi}{x+y}. \tag{.1}$$

Continuity of the mechanism $\mathcal{M}$ implies $\alpha_E^A(x) = \alpha = \alpha_E^A(y)$. Using (.1) and $\alpha_E^A(\sigma^2) = (1 + \lambda)\alpha^F\left(\sigma^2\right)$, it is easy to verify that $x = y$, which provides a contradiction with the starting hypothesis that $\mathcal{P} \subset \Sigma$. Using the same logic one also shows that, as long as the optimal mechanisms features pooling in a subset of Σ, this subset cannot be the union of multiple disjoint intervals (all strictly inside the support Σ).

Hence, if $\mathcal{M}_E^*$ features some pooling, there are only three possible cases to be considered:

(1) $\mathcal{P} = [1 - \Delta, x]$ with $1 - \Delta < x \leq 1 + \Delta$;

(2) $\mathcal{P} = [x, 1 + \Delta]$ with $1 - \Delta \leq x < 1 + \Delta$;

(3) $\mathcal{P} = [x_1, y_1] \cup [x_2, y_2]$ with $1 - \Delta = x_1 < y_1 < x_2 < y_2 = 1 + \Delta$.

Consider first case (1). The investor's maximization problem is

$$\max_{\mathcal{P},\alpha} \mathcal{W}(\mathcal{P}, \alpha) \equiv \max_{x,\alpha} \left\{ \int_{1-\Delta}^x \alpha \left[\pi - \frac{\alpha\sigma^2}{2}\right] d\sigma^2 + \int_x^{1+\Delta} \alpha_E^A(\sigma^2) \left[\pi - \frac{\alpha_E^A(\sigma^2)\sigma^2}{2}\right] d\sigma^2 \right\}.$$

The first-order condition with respect to α is

$$\alpha = \frac{\pi}{\mathbb{E}\left[\sigma^2 | \sigma^2 \leq x\right]} \equiv \frac{2\pi}{1 - \Delta + x}. \tag{.2}$$

Continuity of the mechanism then implies

$$\alpha_E^A(x) = \frac{\pi}{\mathbb{E}\left[\sigma^2 | \sigma^2 \leq x\right]},$$

which yields

$$x_E^* = \frac{1+\lambda}{1-\lambda}\left(1 - \Delta\right), \tag{.3}$$

and thus

$$\alpha_E^*(\sigma^2) = \begin{cases} \frac{\pi(1-\lambda)}{1-\Delta} & \Leftrightarrow \quad \sigma^2 \leq x_E^* \\ \frac{\pi(1+\lambda)}{\sigma^2} & \Leftrightarrow \quad \sigma^2 > x_E^* \end{cases} . \tag{.4}$$

Finally, $1-\Delta < x_E^* \leq 1+\Delta$ implies

$$1-\Delta < \frac{1+\lambda}{1-\lambda}(1-\Delta) \leq 1+\Delta,$$

which requires $\lambda \leq \Delta$. Notice that $x < 1+\Delta$ when $\lambda < \Delta$, so that $\mathcal{P} = [1-\Delta, x] \subset \Sigma$. By contrast, $\mathcal{P} = \Sigma$ when $\lambda \geq \Delta$.

Next, we show that an optimal contract cannot satisfies the properties stated in case (2). The proof of this claim is by contradiction. Under case (2), the investor's maximization problem is

$$\max_{\mathcal{P},\alpha} \mathcal{W}(\mathcal{P},\alpha) \equiv \max_{x,\alpha} \left\{ \int_x^{1+\Delta} \alpha \left[\pi - \frac{\alpha\sigma^2}{2}\right] d\sigma^2 + \int_{1-\Delta}^{x} \alpha_E^A(\sigma^2) \left[\pi - \frac{\alpha_E^A(\sigma^2)\sigma^2}{2}\right] d\sigma^2 \right\}.$$

The first-order condition with respect to α is

$$\alpha = \frac{\pi}{\mathbb{E}\left[\sigma^2|\sigma^2 \geq x\right]} \equiv \frac{2\pi}{x+1+\Delta}.$$

Continuity of the mechanism then implies

$$\alpha_E^A(x) = \frac{\pi}{\mathbb{E}\left[\sigma^2|\sigma^2 \geq x\right]},$$

yielding

$$x = \frac{1+\lambda}{1-\lambda}(1+\Delta), \tag{.5}$$

Notice that $\lambda > 0$ implies $x > 1+\Delta$, which contradicts the starting hypothesis $\mathcal{P} \subset \Sigma$. Therefore, an optimal asset allocation cannot satisfy the properties stated in case (2).

Finally, consider case (3). In this scenario the investor pools in two disjoint intervals, say $\mathcal{P}_1$=$[1-\Delta, x_1]$ and $\mathcal{P}_2 = [x_2, 1+\Delta]$, with $1-\Delta < x_1 < x_2 = 1+\Delta$. Let $\mathcal{P}$ =$(\mathcal{P}_1, \mathcal{P}_2)$, $\alpha^{\mathcal{P}} = (\alpha^{\mathcal{P}_1}, \alpha^{\mathcal{P}_2})$ and $\mathbf{x} = (x_1, x_2)$. The investor's maximization problem is

$$\begin{aligned} \max_{\mathcal{P},\alpha^{\mathcal{P}}} \mathcal{W}(\mathcal{P},\alpha^{\mathcal{P}}) \equiv \max_{\mathbf{x},\alpha^{\mathcal{P}}} \Bigg\{ & \int_{1-\Delta}^{x_1} \alpha^{\mathcal{P}_1} \left[\pi - \frac{\alpha^{\mathcal{P}_1}\sigma^2}{2}\right] d\sigma^2 + \\ & + \int_{x_1}^{x_2} \alpha_E^A(\sigma^2) \left[\pi - \frac{\alpha_E^A(\sigma^2)\sigma^2}{2}\right] d\sigma^2 + \int_{x_2}^{1+\Delta} \alpha^{\mathcal{P}_2} \left[\pi - \frac{\alpha^{\mathcal{P}_2}\sigma^2}{2}\right] d\sigma^2 \Bigg\}, \end{aligned}$$

where $\alpha^{\mathcal{P}_1}$ and $\alpha^{\mathcal{P}_2}$ are the shares of wealth invested into the risky asset within the pooling regions $\mathcal{P}_1$ and $\mathcal{P}_2$, respectively. Optimality then requires

$$\alpha^{\mathcal{P}_k} = \frac{\pi}{\mathbb{E}\left[\sigma^2|\sigma^2 \in \mathcal{P}_k\right]} \quad k = 1,2.$$

Continuity of the mechanism then implies

$$\frac{\pi}{\mathbb{E}\left[\sigma^2|\sigma^2 \le x_1\right]} = \alpha_E^A(x_1),$$

$$\frac{\pi}{\mathbb{E}\left[\sigma^2|\sigma^2 \ge x_2\right]} = \alpha_E^A(x_2).$$

Solving for x_1 and x_2

$$x_1 = \frac{1+\lambda}{1-\lambda}\left(1-\Delta\right),$$

$$x_2 = \frac{1+\lambda}{1-\lambda}\left(1+\Delta\right) > 1+\Delta,$$

which is a contradiction. Therefore, an optimal asset allocation cannot satisfy the properties stated in case (3).

In order to complete the characterization of the optimal mechanism we need to show that the investor never gains from full delegation — i.e., $\mathcal{P} = \varnothing$ cannot be an optimum. Specifically, for any asset allocation rule that satisfies (1) the following holds

$$\left.\frac{\partial \mathcal{W}(x, \alpha(x))}{\partial x}\right|_{x=1-\Delta} = \frac{\gamma\lambda^2\pi^2}{2\left(1-\Delta\right)} > 0.$$

where

$$\alpha(x) \equiv \frac{2\pi}{1-\Delta+x}.$$

But this directly implies $x > 1-\Delta$. Finally, notice that $\mathcal{W}(x,\alpha)$ is concave in α. Substituting for $\alpha_E^*\left(x\right)$ we have

$$\left.\frac{\partial^2 \mathcal{W}(x, \alpha(x))}{\partial x^2}\right|_{x=x_E^*} = -\frac{\gamma}{2}\pi^2 \frac{\left(1-\lambda\right)^3}{\left(1-\Delta\right)^2\left(1+\lambda\right)}\left(2\lambda+1\right) < 0,$$

which completes the proof. ■

Proof of Lemma 1. The proof of the lemma is by contradiction.

To begin with, we show that there exists no symmetric equilibrium where both investors offer a mechanism $\mathcal{M}$ that requires pooling in the subset $\mathcal{P} \subset \Sigma$. Let $\mathcal{P} \equiv [x, y]$, with $1-\Delta < x < y < 1+\Delta$, be one of the (disjoint) regions in which both investors pool at equilibrium — i.e., the subset of Σ in which each investor chooses α for every $\sigma^2 \in \mathcal{P}$. Hence, outside $\mathcal{P}$ there must exist some intervals of Σ in which both investors delegate their portfolio choices to the advisor, thus allocating a fraction $\alpha_N^A(\sigma^2)$ of their wealth into the risky asset.

Continuity of the mechanism $\mathcal{M}$ then implies $\alpha_N^A(x) = \alpha_N^A(y) = \alpha$, where optimality requires

$$\alpha = \frac{\pi}{\mathbb{E}\left[\sigma^2|\sigma^2 \in \mathcal{P}\right]}. \tag{.6}$$

Using (.6) and the expression for $\alpha_N^A(\sigma^2)$ this system of equations can be rewritten as

$$\alpha_N^A(x) = \frac{2\pi}{x+y},$$

$$\alpha_N^A(y) = \frac{2\pi}{x+y},$$

whose unique solution requires $x = y$, which contradicts $\mathcal{P} \subset \Sigma$.

Next, we prove the remaining part of the Lemma. First, notice that the above argument rules out the possibility of having multiple (disjoint) delegation regions strictly contained in Σ. Otherwise, we should have at least one pooling region strictly contained in Σ. However, this does not exclude the existence of a single delegation region strictly inside Σ, surrounded by two pooling regions. In what follows we show that this cannot be possible as well. The proof is again by contradiction. Suppose that such a symmetric equilibrium exists. Accordingly, let $\mathcal{P}_1 \equiv [1-\Delta, x]$ and $\mathcal{P}_2 \equiv [y, 1+\Delta]$, with $1-\Delta < x < y < 1+\Delta$, be the two regions where both investors pool — i.e., the subsets of Σ in which each investor chooses $\alpha^{\mathcal{P}_1}$ for every $\sigma^2 \in \mathcal{P}_1$ and $\alpha^{\mathcal{P}_2}$ for every $\sigma^2 \in \mathcal{P}_2$. On the contrary, delegation takes place in the interior $[x, y]$.

Continuity of the mechanism $\mathcal{M}$ then implies $\alpha_N^A(x) = \alpha^{\mathcal{P}_1}$ and $\alpha_N^A(y) = \alpha^{\mathcal{P}_2}$ — where, optimality of the mechanism requires

$$\alpha^{\mathcal{P}_1} = \frac{\pi}{\mathbb{E}\left[\sigma^2|\sigma^2 \in \mathcal{P}_1\right]},$$

$$\alpha^{\mathcal{P}_2} = \frac{\pi}{\mathbb{E}\left[\sigma^2|\sigma^2 \in \mathcal{P}_2\right]}.$$

Hence

$$x = \frac{1+\lambda}{1-\lambda+2\theta}(1-\Delta), \qquad y = \frac{1+\lambda}{1-\lambda+2\theta}(1+\Delta),$$

which clearly contradict the starting hypothesis that $1-\Delta < x$ and $y < 1+\Delta$. ■

Proof of Proposition 2. The proof of the proposition is structured as follows. First, we characterize the properties that a symmetric (candidate) equilibrium where both investors pool for low values of σ^2 and delegate otherwise need to satisfy. Second, we show that, in the region of parameters under consideration, this outcome is immune from unilateral deviations within the class of continuous mechanisms. Third, we show that there exists a non-empty region of parameters where there exists an equilibrium with full pooling. Finally, we argue that within the regions of parameters under consideration there are no other symmetric equilibria.

Suppose that both investors offer a mechanism $\mathcal{M}_N^* = \left\{\alpha_N^*(\sigma^2)\right\}_{\sigma^2\in\Sigma}$ that entails a pooling allocation in the subset $\mathcal{P}_N^* \equiv [1-\Delta, x_N^*] \subseteq \Sigma$, with $1-\Delta < x_N^* < 1+\Delta$, and a separating one for $\sigma^2 > x_N^*$ — i.e.,

$$\alpha_N^*(\sigma^2) = \begin{cases} \alpha_N^* & \Leftrightarrow \quad \sigma^2 \le x_N^* \\ \alpha_N^A(\sigma^2) & \Leftrightarrow \quad \sigma^2 > x_N^* \end{cases}.$$

The first-order condition identifying α_N^* is

$$\alpha_N^* = \frac{\pi}{\mathbb{E}\left[\sigma^2|\sigma^2 \in \mathcal{P}_N^*\right]}, \tag{.7}$$

While continuity of $\mathcal{M}_N^*$ requires $\alpha_N^* = \alpha_N^A(x_N^*)$. Substituting (.7) and the expression for $\alpha_N^A(\sigma^2)$ into this equation, x_N^* solves

$$\frac{x_N^*}{\mathbb{E}\left[\sigma^2|\sigma^2 \leq x_N^*\right]} = \frac{1+\lambda}{1+\theta},$$

yielding

$$x_N^* = \frac{1+\lambda}{1-\lambda+2\theta}\left(1-\Delta\right),$$

and thus

$$\alpha_N^* = \pi\frac{1-\lambda+2\theta}{\left(1+\theta\right)\left(1-\Delta\right)}.$$

Notice that $x_N^* \in (1-\Delta, 1+\Delta)$ in the region of parameters under consideration. Following the approach of Proposition 1 concavity of the investors' expected utility at $\left(x_N^*, \alpha_N^*(\sigma^2)\right)$ can be easily checked. Hence, $\theta > (\lambda - \Delta)/(1+\Delta)$ and $\lambda < \Delta$ are necessary conditions for such a symmetric equilibrium to exist. In the following we show that they are also sufficient.

Next, we show that, within the class of continuous mechanisms and incentive compatible mechanisms, there are no profitable deviations from the symmetric outcome characterized above. The proof is developed in the following steps, where it is assumed (without loss of generality) that investor 1 sticks to the equilibrium behavior.

Step 1. Consider first the class of deviations where investor 2 offers a mechanism $\mathcal{M}_2$ such that $\alpha_2(.)$ is constant in a neighborhood of $\sigma^2 = 1-\Delta$ — i.e., there exists a non-empty neighborhood of $1-\Delta$, say $\mathcal{B}(1-\Delta)$, such that $\mathcal{B}(1-\Delta) \subseteq \Sigma$ and $\dot{\alpha}_2\left(\sigma^2\right) = 0$ for every $\sigma^2 \in \mathcal{B}(1-\Delta)$.

1.A. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails pooling only in region $\mathcal{P}_2 \equiv [1-\Delta, x]$, with $1-\Delta < x < 1+\Delta$ and $x \neq x_N^*$.

Showing that this deviation is unprofitable in the region of parameters under consideration is straightforward. Indeed, continuity of the optimal mechanism implies $x = x_N^*$. This is immediate for $x > x_N^*$. By contrast, for $x < x_N^*$ continuity of $\mathcal{M}_2$ requires

$$\frac{\pi}{\mathbb{E}[\sigma^2|\sigma^2 \leq x]} = \alpha_E^A(x) - \theta\alpha_N^*,$$

which has only one solution in Σ equal to x_N^*. A contradiction.

1.B. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails pooling in regions $\mathcal{P}_2 \equiv [1-\Delta, x]$ and $\mathcal{P}_2' = [y, 1+\Delta]$, with $y > x \geq x_N^*$.

Continuity of $\mathcal{M}_2$ requires

$$\frac{\pi}{\mathbb{E}[\sigma^2|\sigma^2 \leq x]} = \alpha_N^A\left(x\right) \quad \Leftrightarrow \quad x = x_N^*,$$

$$\frac{\pi}{\mathbb{E}[\sigma^2|\sigma^2 \geq y]} = \alpha_N^A\left(y\right) \quad \Leftrightarrow \quad y = \frac{1+\lambda}{1-\lambda+2\theta}\left(1+\Delta\right).$$

But, in the region of parameters under consideration it is easy to verify that $y > 1+\Delta$. A contradiction.

1.C. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails pooling in regions $\mathcal{P}_2 \equiv [1-\Delta, x]$ and $\mathcal{P}_2' = [y, 1+\Delta]$, with $1+\Delta > y > x_N^* > x > 1-\Delta$.

In this case the same contradiction obtained in step **1.B** obtains.

1.D. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails pooling in regions $\mathcal{P}_2 \equiv [1-\Delta, x]$ and $\mathcal{P}_2' = [y, 1+\Delta]$, with $1+\Delta > x_N^* \geq y > x > 1-\Delta$.

Continuity of $\mathcal{M}_2$ requires

$$\frac{\pi}{\mathbb{E}[\sigma^2|\sigma^2 \leq x]} = \alpha_E^A(x) - \theta\alpha_N^*,$$

$$\frac{\pi}{\mathbb{E}[\sigma^2|\sigma^2 \geq y]} = \alpha_E^A(y) - \theta\alpha_N^*.$$

Hence

$$\frac{2}{1-\Delta+x} = \frac{1+\lambda}{x} - \theta\frac{1-\lambda+2\theta}{(1+\theta)(1-\Delta)}, \tag{.8}$$

$$\frac{2}{1+\Delta+y} = \frac{1+\lambda}{y} - \theta\frac{1-\lambda+2\theta}{(1+\theta)(1-\Delta)}. \tag{.9}$$

Notice that, in the region of parameters under consideration, (.8) has two solutions: one negative and one equal to x_N^*. A contradiction with the initial assumption that $x_N^* \geq y > x$.

1.E. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails full pooling. In order to show that this cannot be a best reply, consider the case where investor 2 deviates by using a strategy like the ones considered in **1.A**. Clearly, full pooling is a degenerated form of this class of strategy, where $x = 1+\Delta$. But, this corner solution can be optimal if and only if the derivative of the unconstrained maximization problem of investor 2 with respect to x is non negative at $x = 1+\Delta$. Recall that if $x = 1+\Delta$, then optimality requires investor 2 to invest π into the risky asset for every σ^2. Hence, the derivative of investor 2's expected utility with respect to x evaluated at $x = 1+\Delta$ is

$$\gamma\left[\pi - \alpha_N^A(1+\Delta)\right] \times \left[\pi - \tfrac{1}{2}\left[\alpha_N^A(1+\Delta)+\pi\right](1+\Delta)\right] = \\ -\gamma\pi^2\frac{\Delta-\lambda+\theta+\Delta\theta}{(1+\theta)(1+\Delta)} \times \frac{\Delta(1+\theta)+\lambda-\theta}{2(1+\theta)},$$

which is strictly negative in the region of parameters under consideration. A contradiction.

1.F. Finally, using the same arguments developed in the proof of cases **1.B**, **1.C,** and **1.D** it is easy to verify that deviations starting with a pooling allocation at $1-\Delta$ and involving at least two disjoint separation regions are not profitable.

Step 2. Consider now the class of deviations where investor 2 offers a mechanism $\mathcal{M}_2$ such that $\alpha_2(.)$ is (strictly) decreasing in a neighborhood of $\sigma^2 = 1-\Delta$ — i.e., there exists a non-empty neighborhood of $1-\Delta$, say $\mathcal{B}(1-\Delta)$, such that $\mathcal{B}(1-\Delta) \subseteq \Sigma$ and $\dot{\alpha}_2(\sigma^2) < 0$ for every $\sigma^2 \in \mathcal{B}(1-\Delta)$.

2.A. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails pooling only in region $\mathcal{P}_2 \equiv [x, 1+\Delta]$, with $1-\Delta < x_N^* \leq x < 1+\Delta$.

Continuity of $\mathcal{M}_2$ requires

$$\alpha_N^A(x) = \frac{\pi}{\mathbb{E}[\sigma^2|\sigma^2 \geq x]} \quad \Leftrightarrow \quad x = \frac{1+\lambda}{1-\lambda+2\theta}(1+\Delta).$$

By construction $x < 1 + \Delta$, which would imply $\lambda < \theta$. A contradiction.

2.B. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails pooling only in region $\mathcal{P}_2 \equiv [x, 1 + \Delta]$, with $1 - \Delta < x < x_N^* < 1 + \Delta$.

Continuity of $\mathcal{M}_2$ requires

$$\frac{\pi}{\mathbb{E}[\sigma^2 | \sigma^2 \geq x]} = \alpha_E^A(x) - \theta \alpha_N^*,$$

that is

$$\frac{2\pi}{x + 1 + \Delta} = (1 + \lambda) \frac{\pi}{x} - \theta \alpha_N^*, \tag{.10}$$

Let

$$\Phi(z) \equiv (1 + \lambda) \frac{\pi}{z} - \frac{2\pi}{z + 1 + \Delta}.$$

Condition (.10) then rewrites as $\Phi(x) = \theta \alpha_N^*$. Notice that

$$\Phi(1 - \Delta) = \pi \frac{\lambda + \Delta}{1 - \Delta} > \Phi(1 + \Delta) = \frac{\pi \lambda}{1 + \Delta} > 0,$$

and

$$\Phi'(z) = -\pi \frac{(1 + \Delta)^2 (1 + \lambda) - z^2 (1 - \lambda) + 2z (1 + \Delta)(1 + \lambda)}{z^2 (\Delta + 1 + z)^2},$$

where it can be verified that

$$(1 + \Delta)^2 (1 + \lambda) - z^2 (1 - \lambda) + 2z (1 + \Delta)(1 + \lambda) > 0 \quad \forall z \in \Sigma.$$

Hence, $\Phi'(z) < 0$ in Σ. Taken together, these conditions imply that $\Phi(x) > 0$ in Σ.

Next, note that for $\theta < 0$, (.10) has no solution, which yields a contradiction.

2.C. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails pooling only in region $\mathcal{P}_2 \equiv [x, y]$, with $1 - \Delta < x < y < 1 + \Delta$.

First, it is straightforward to show that continuity of the mechanism (together with optimality) rules out deviations such that $y > x > x_N^*$ and $x_N^* > y > x$.

Next, consider a deviation such that $x < x_N^* < y$. Continuity of $\mathcal{M}_2$ imply

$$\frac{\pi}{\mathbb{E}[\sigma^2 | \sigma^2 \in \mathcal{P}_2]} = \alpha_E^A(x) - \theta \alpha_N^*,$$

$$\frac{\pi}{\mathbb{E}[\sigma^2 | \sigma^2 \in \mathcal{P}_2]} = \alpha_N^A(y).$$

Hence,

$$\frac{2\pi}{x + y} = (1 + \lambda) \frac{\pi}{x} - \theta \alpha_N^*,$$

$$\frac{2y}{x + y} = \frac{1 + \lambda}{1 + \theta}.$$

The solution of this system of equations is

$$x = \frac{2\lambda - \theta(1-\lambda)}{(1-\lambda+2\theta)\theta}(1-\Delta),$$

$$y = \frac{(2\lambda - \theta(1-\lambda))(1+\lambda)}{(1-\lambda+2\theta)^2\theta}(1-\Delta).$$

Notice that

$$x - (1-\Delta) = \frac{2(\lambda-\theta)(1+\theta)}{(1-\lambda+2\theta)\theta}(1-\Delta) < 0 \quad \Leftrightarrow \quad \theta < 0.$$

Hence, for $\theta < 0$, this yields a contradiction in the region of parameters under consideration.

2.D. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails full delegation.

Notice that, given the mechanism offered by investor 1, the advisor's ideal point over investor 2's asset allocation choice is

$$\alpha_2^A(\sigma^2) = \begin{cases} \alpha_N^A(\sigma^2) & \Leftrightarrow \quad \sigma^2 \geq x_N^* \\ \alpha_E^A(\sigma^2) - \theta\alpha_N^* & \Leftrightarrow \quad \sigma^2 < x_N^* \end{cases}.$$

Hence, $\theta \leq 0$ implies $\alpha_2^A(\sigma^2) > \alpha^F(\sigma^2)$. Then, using the same logic of the proof of Proposition 1, it follows that for investor 2 it is optimal to pool in a non-empty subset of Σ. A contradiction.

2.E. Finally, using the same arguments developed in the proof of cases **2.A, 2.B,** and **2.C**, it can be verified that deviations starting with a separating at $\sigma^2 = 1-\Delta$ and involving at least two disjoint pooling regions are not profitable.

We now turn to characterize the region of parameters where there exists pooling equilibrium. Recall that in an equilibrium with full pooling both investors choose $\alpha_N^*(\sigma^2) = \pi$ regardless of the advisor's reports. We must then show that there exists a region of parameters where there are no profitable deviations from this outcome. As before, assume (without loss of generality) that investor 1 pools — i.e., $\alpha_1 = \pi \ \forall m \in \Sigma$. Consider a deviation by investor 2 such that

$$\alpha_2(\sigma^2) = \begin{cases} \overline{\alpha}_2 & \Leftrightarrow \quad \sigma^2 < \underline{x} \\ \alpha_2^A(\sigma^2) & \Leftrightarrow \quad \sigma^2 \in [\underline{x}, \overline{x}] \\ \underline{\alpha}_2 & \Leftrightarrow \quad \sigma^2 > \overline{x} \end{cases}, \tag{.11}$$

where $1+\Delta \geq \overline{x} \geq \underline{x} \geq 1-\Delta$. Notice that incentive compatibility implies $\overline{\alpha}_2 > \underline{\alpha}_2$ and

$$\alpha_2^A(\sigma^2) = \alpha_E^A(\sigma^2) - \theta\pi.$$

Notice that whenever $\underline{x} \geq 1+\Delta$, investor 2's best reaction entails full pooling. If this is not the case, one must have that the solution of investor 2's maximization problem requires $1+\Delta \geq \overline{x} > \underline{x} \geq 1-\Delta$. In what follows we show that in the region of parameters where $\theta \leq (\lambda-\Delta)/(1+\Delta)$ this is not possible.

The derivative of investor 2's expected utility with respect to $\underline{x}$ is

$$\gamma\left[\overline{\alpha}_2-\alpha_2^A(\underline{x})\right]\times\left[\pi-\tfrac{1}{2}\left[\alpha_2^A\left(\underline{x}\right)+\overline{\alpha}_2\right]\underline{x}\right], \tag{.12}$$

where

$$\overline{\alpha}_2=\frac{\pi}{\mathbb{E}\left[\sigma^2|\sigma^2\leq\underline{x}\right]}.$$

Evaluating (.12) at $\underline{x}=1+\Delta$, so that $\overline{\alpha}_2=\pi$, this derivative can be rewritten as

$$\gamma\pi^2\left[\frac{\lambda-\Delta\left(1+\theta\right)-\theta}{1+\Delta}\right]\times\left[\frac{\lambda+\Delta\left(1-\theta\right)-\theta}{2}\right]. \tag{.13}$$

By the same token, the derivative of investor 2's expected utility with respect to $\overline{x}$ is

$$\gamma\left[\alpha_2^A(\overline{x})-\underline{\alpha}_2\right]\times\left[\pi-\tfrac{1}{2}\left[\alpha_2^A\left(\overline{x}\right)+\underline{\alpha}_2\right]\overline{x}\right], \tag{.14}$$

where

$$\underline{\alpha}_2=\frac{\pi}{\mathbb{E}\left[\sigma^2|\sigma^2\geq\overline{x}\right]}.$$

Evaluating (.14) at $\overline{x}=1-\Delta$ so that $\underline{\alpha}_2=\pi$, this derivative can be rewritten as

$$-\gamma\pi^2\left[\frac{\lambda-\theta\left(1+\Delta\right)+\Delta}{1-\Delta}\right]\times\left[\frac{\lambda-\Delta\left(1-\theta\right)-\theta}{2}\right]. \tag{.15}$$

In this region of parameters under consideration, the expression in equation (.13) is positive. By concavity of investor 2's objective function (which can be immediately checked) it then follows that investor 2's best reply to full pooling by investor 1 is full pooling, which shows that full pooling is an equilibrium in this region.

Moreover, notice that a strategy for investor 2 that requires a pooling allocation only when $\sigma^2\in int\Sigma$ is not feasible: this would indeed contradict continuity of the optimal mechanisms. Hence, in the region of parameters under consideration there is a unique symmetric equilibrium where both investors pool. This result, together with the first part of the proof also implies that in each of the two regions of parameters identified by the statement of the proposition there is a unique symmetric equilibrium. ■

Proof of Proposition 3. The proof of the proposition is structured as follows. As before, we first characterize the properties that a symmetric (candidate) equilibrium where both investors pool for high values of σ^2 and delegate otherwise need to satisfy. Second, we show that, in the region of parameters under consideration, this outcome is immune from unilateral deviations within the class of continuous mechanisms. Third, we show that there exists a non-empty region of parameters where there exists an equilibrium with full pooling. Finally, we argue that within the regions of parameters under consideration there are no other symmetric equilibria.

To begin with, notice that when $\lambda=\theta$ there is a unique symmetric equilibrium with full delegation. Second, it can be check that the proof of Proposition 2 easily extends to the case where $\lambda>\theta>0$. Hence, for brevity we will focus on the novel type of equilibrium that emerges when $\theta>\lambda$.

Suppose that both investors offer a mechanism $\mathcal{M}_N^*\equiv\left\{\alpha_N^*(\sigma^2)\right\}_{\sigma^2\in\Sigma}$ that entails a pooling allocation in the subset $\mathcal{P}_N^* \equiv [x_N^*, 1+\Delta] \subseteq \Sigma$, with $1-\Delta < x_N^* < 1+\Delta$, and a separating one for $\sigma^2 < x_N^*$ — i.e.,

$$\alpha_N^*(\sigma^2) = \begin{cases} \alpha_N^A(\sigma^2) & \Leftrightarrow \quad \sigma^2 \leq x_N^* \\ \alpha_N^* & \Leftrightarrow \quad \sigma^2 > x_N^* \end{cases}.$$

Form the optimality conditions of the investors' maximization problem, it follows that

$$\alpha_N^* = \frac{\pi}{\mathbb{E}\left[\sigma^2 | \sigma^2 \in \mathcal{P}\right]} \equiv \frac{2\pi}{x_N^* + 1 + \Delta}, \tag{.16}$$

while continuity of the mechanism implies that $\alpha_N^* = \alpha_N^A(x_N^*)$. Substituting (.16) and the expression for $\alpha_N^A(\sigma^2)$, this equation rewrites as

$$\frac{2\pi}{x_N^* + 1 + \Delta} - \alpha_N^A(x^*) = 0,$$

whose unique solution yields

$$x_N^* = \frac{1+\lambda}{1-\lambda+2\theta}\left(1+\Delta\right).$$

Therefore, the pooling allocation is

$$\alpha_N^* = \pi \frac{1-\lambda+2\theta}{\left(1+\theta\right)\left(1+\Delta\right)}.$$

Since $\lambda < 1$, it follows that $x_N^* \in (1-\Delta, 1+\Delta)$ in the region of parameters where $\lambda < \theta < \left(\lambda+\Delta\right)/\left(1-\Delta\right)$. As before, it can be shown that concavity of the investors' expected utility holds at (x_N^*, α_N^*) within the region of parameters under consideration. Hence, $\lambda < \theta < \left(\lambda+\Delta\right)/\left(1-\Delta\right)$ is a necessary condition for such a symmetric equilibrium to exist. In the following we show that they are also sufficient.

Next, we show that, within the class of continuous mechanisms and incentive compatible mechanisms, there are no profitable deviations from the symmetric outcome characterized above. The proof is developed in the following steps, where it is assumed (without loss of generality) that investor 1 sticks to the equilibrium behavior.

Step 1. Consider first the class of deviations where investor 2 offers a mechanism $\mathcal{M}_2$ such that $\alpha_2(.)$ is constant in a neighborhood of $\sigma^2 = 1+\Delta$.

1.A. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails pooling only in region $\mathcal{P}_2 \equiv [x, 1+\Delta]$, with $1-\Delta < x_N^* < 1+\Delta$ and $x \neq x_N^*$.

Showing that this deviation is unprofitable in the region of parameters under consideration is straightforward. Indeed, continuity of the optimal mechanism implies $x = x_N^*$. This is immediate for $x < x_N^*$. By contrast, for $x > x_N^*$ continuity of of $\mathcal{M}_2$ requires

$$\frac{\pi}{\mathbb{E}[\sigma^2 | \sigma^2 \geq x]} = \alpha_E^A(x) - \theta\alpha_N^*,$$

which has only one solution in Σ equal to x_N^*. A contradiction.

1.B. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails pooling in regions

$\mathcal{P}_2 \equiv [1-\Delta, x]$ and $\mathcal{P}_2' = [y, 1+\Delta]$, with $x < y \leq x_N^*$.

By continuity, the optimal mechanism $\mathcal{M}_2$ must satisfy

$$\frac{2\pi}{1-\Delta+x} = \alpha_N^A(x) \quad \Leftrightarrow \quad x = \frac{1+\lambda}{1-\lambda+2\theta}(1-\Delta),$$

$$\frac{2\pi}{1+\Delta+y} = \alpha_N^A(y) \quad \Leftrightarrow \quad y = x_N^*.$$

But, in the region of parameters under consideration it is easy to verify that $x < 1-\Delta$: a contradiction.

1.C. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails pooling in regions $\mathcal{P}_2 \equiv [1-\Delta, x]$ and $\mathcal{P}_2' = [y, 1+\Delta]$, with $1+\Delta > y > x_N^* > x > 1-\Delta$.

In this case the same contradiction obtained in step **1.B** obtains**.**

1.D. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails a pool allocation in regions $\mathcal{P}_2 \equiv [1-\Delta, x]$ and $\mathcal{P}_2' = [y, 1+\Delta]$, with $1+\Delta > y > x \geq x_N^* > 1-\Delta$.

By continuity, the optimal mechanism $\mathcal{M}_2$ must satisfy

$$\frac{2\pi}{\mathbb{E}[\sigma^2|\sigma^2 \leq x]} = \alpha_E^A(x) - \theta\alpha_N^*,$$

$$\frac{2\pi}{\mathbb{E}[\sigma^2|\sigma^2 \geq y]} = \alpha_E^A(y) - \theta\alpha_N^*.$$

Hence, x and y must solve

$$\frac{2}{1-\Delta+x} = \frac{1+\lambda}{x} - \theta\frac{1-\lambda+2\theta}{(1+\theta)(1+\Delta)},$$

$$\frac{2}{1+\Delta+y} = \frac{1+\lambda}{y} - \theta\frac{1-\lambda+2\theta}{(1+\theta)(1+\Delta)}.$$

Notice that, in the region of parameters under consideration, the latter equation has a unique positive solution $y = x_N^*$. A contradiction with the initial assumption that $y > x \geq x_N^*$.

1.E. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails full pooling. In order to show that this is not optimal, consider the case where investor 2 deviates by using a strategy like the ones considered in **1.A**. Clearly, full pooling is a degenerated form of this class of strategy, where $x = 1-\Delta$. But, this corner solution can be optimal if and only if the derivative of the unconstrained maximization problem of investor 2 with respect to x is non positive at $x = 1-\Delta$. Recall that if $x = 1-\Delta$, then optimality requires $\alpha_2^* = \pi$ for every σ^2. The derivative of investor 2's expected utility with respect to x evaluated at $x = 1-\Delta$ is

$$\gamma\left[\alpha_N^A(1-\Delta) - \pi\right] \times \left[\pi - \tfrac{1}{2}\left[\alpha_N^A(1-\Delta) + \pi\right](1-\Delta)\right] = \gamma\pi^2\frac{(\Delta+\lambda-\theta+\Delta\theta)}{(1+\theta)(1-\Delta)}\frac{(\Delta(1+\theta)-\lambda+\theta)}{2(1+\theta)} > 0,$$

which is strictly in the region of parameters under consideration: a contradiction.

1.F. Using the same arguments developed in the proof of cases **1.B, 1.C** and **1.D**, it is easy to verify that deviations starting with a pooling at $1+\Delta$ and involving at least two disjoint separation regions are not profitable.

Step 2. Consider now the class of deviations where investor 2 offers a mechanism $\mathcal{M}_2 = \left\{\alpha_2\left(\sigma^2\right)\right\}_{\sigma^2\in\Sigma}$ such that $\alpha_2(.)$ is (strictly) decreasing in a neighborhood of $\sigma^2 = 1+\Delta$ — i.e., there exists a non-empty neighborhood of $1+\Delta$, say $\mathcal{B}(1+\Delta)$, such that $\mathcal{B}(1+\Delta) \subseteq \Sigma$ and $\dot{\alpha}_2\left(\sigma^2\right) < 0$ for every $\sigma^2 \in \mathcal{B}(1+\Delta)$.

2.A. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails pooling only in region $\mathcal{P}_2 \equiv [1-\Delta, x]$, with $1-\Delta < x \le x_N^* < 1+\Delta$.

Continuity of the mechanism, together with optimality, imply

$$\alpha_N^A(x) = \frac{2\pi}{1-\Delta+x} \quad \Leftrightarrow \quad x = \frac{1+\lambda}{1-\lambda+2\theta}(1-\Delta).$$

By construction $x > 1-\Delta$, which requires $\lambda > \theta$. But, this condition is not met in the region of parameters under consideration.

2.B. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails pooling only in region $\mathcal{P}_2 \equiv [1-\Delta, x]$, with $1-\Delta < x_N^* < x < 1+\Delta$.

Continuity of the mechanism, together with optimality, imply

$$\alpha_E^A(x) - \theta\alpha_N^* = \frac{2\pi}{1-\Delta+x}. \tag{.17}$$

Define

$$\Phi(z) \equiv \alpha_E^A(z) - \frac{2\pi}{1-\Delta+z}.$$

Condition (.17) rewrites as $\Phi(x) = \theta\alpha_N^*$. Notice that

$$\Phi(1-\Delta) = \frac{\pi\lambda}{1-\Delta} > 0, \qquad \Phi(1+\Delta) = \pi\frac{\lambda-\Delta}{1+\Delta},$$

$$\Phi'(z) = -\pi\frac{(1-\Delta)^2(1+\lambda) - z^2(1-\lambda) + 2z(1-\Delta)(1+\lambda)}{z^2(z+1-\Delta)^2},$$

and

$$\Phi(x_N^*) - \theta\alpha_N^* = -\frac{(1-\lambda+2\theta)^2\Delta\pi}{(1+\theta(1-\Delta)+\Delta\lambda)(1+\theta)(1+\Delta)} < 0.$$

Moreover, $\Phi(1+\Delta) > 0$ and $\Phi'(z) < 0$ for every $z \in \Sigma$ if $\lambda > \Delta$. Hence, $\Phi(x_N^*) < \theta\alpha_N^*$ directly implies that $x_N^* > x$ for $\lambda > \Delta$: a contradiction. Next, suppose that $\lambda \le \Delta$. In this region of parameters it is easy to verify that $\Phi'(z) = 0$ has a unique solution in Σ (say z^*) with $\Phi'(z) > 0$ if and only if $z > z^*$ and $\Phi(z^*) < 0$. Hence, $\Phi(1+\Delta) \le 0$ together with $\Phi(x_N^*) < \theta\alpha_N^*$, directly imply that $x_N^* > x$ *a fortiori* when $\lambda \le \Delta$: again a contradiction.

2.C. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails pooling only in region $\mathcal{P}_2 \equiv [x, y]$, with $1-\Delta < x < y < 1+\Delta$.

First, it is straightforward to show that continuity of the mechanism (together with optimality) rules out deviations such that $y > x > x_N^*$ and $x_N^* > y > x$.

Next, consider a deviation such that $x < x_N^* < y$. Continuity of $\mathcal{M}_2$, together with optimality, imply

$$\alpha_N^A(x) = \frac{2\pi}{x+y},$$

$$\alpha_E^A(y) - \theta\alpha_N^* = \frac{2\pi}{x+y}.$$

The solution of this system of equations is

$$x = (1+\lambda)(1+\Delta)\frac{2\lambda-\theta+\lambda\theta}{\theta(1-\lambda+2\theta)^2},$$

$$y = (1+\Delta)\frac{2\lambda-\theta+\lambda\theta}{\theta(1-\lambda+2\theta)}.$$

Notice that

$$y - x_N^* = \frac{2(\lambda-\theta)}{(1-\lambda+2\theta)\theta}(1+\Delta) < 0,$$

since $\lambda < \theta$ in the region of parameters under consideration, yielding the desired contradiction.

2.D. Suppose that investor 2 deviates by offering a mechanism $\mathcal{M}_2$ that entails full delegation.

Notice that, given the mechanism offered by investor 1, the advisor's ideal point over investor 2's asset allocation choice is

$$\alpha_2^*(\sigma^2) = \begin{cases} \alpha_N^A(\sigma^2) & \Leftrightarrow \quad \sigma^2 \leq x_N^* \\ \alpha_E^A(\sigma^2) - \theta\alpha_N^* & \Leftrightarrow \quad \sigma^2 > x_N^* \end{cases}.$$

Hence: $\alpha_N^A(\sigma^2) < \alpha^F(\sigma^2)$ since $\theta > \lambda$ and

$$\lambda\alpha^F(x_N^*) < \theta\alpha_N^* \quad \Rightarrow \quad \alpha_E^A(\sigma^2) - \theta\alpha_N^* < \alpha^F(\sigma^2) \quad \forall\sigma^2 > x_N^*.$$

But this implies that $\alpha_2^*(\sigma^2) < \alpha^F(\sigma^2)$ for all $\sigma^2 \in \Sigma$. Then, by the same logic of the proof of Proposition 1, it follows that for investor 2 it is optimal to pool in a non-empty subset of Σ. A contradiction.

2.E. Finally, using the same arguments developed in the proof of cases **2.A**, **2.B** and **2.C,** it is easy to verify that deviations starting with a separating at $\sigma^2 = 1+\Delta$ and involving at least two disjoint pooling regions are not profitable.

The rest of the proof follows the same arguments used in the proof of Proposition 2. ■

Proof of Corollary 1. We prove the result only for the exclusivity benchmark, the proof for the game with non-exclusivity follows exactly the same logic and is omitted for brevity.

Suppose that the investor offers a very simple delegation mechanism to the advisor that requires him to choose the amount of wealth to allocate to the risky asset within the range $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, with

$$\underline{\alpha} = \pi\frac{1+\lambda}{1+\Delta}, \qquad \overline{\alpha} = \pi\frac{1-\lambda}{1-\Delta}.$$

For every σ^2, the advisor's optimization problem is

$$\min_{\alpha\in[\underline{\alpha},\overline{\alpha}]} \frac{1}{2}\left[\alpha-(1+\lambda)\,\alpha^F(\sigma^2)\right]^2.$$

The result then follows immediately since $\alpha_E^A(\sigma^2)\geq\overline{\alpha}$ if and only if $\sigma^2\leq x_E^*$, and $\alpha_E^A(\sigma^2)\geq\underline{\alpha}$ with equality only at $\sigma^2=1+\Delta$. ■

Proof of Proposition 4. To show this result we first need to compute the investors' expected utility with and without exclusivity.

Consider first the exclusivity benchmark. Using the result of Proposition 1 the investor's expected (indirect) utility is

$$\mathcal{W}_E^*=\frac{\gamma}{2\Delta}\left[\int_{1-\Delta}^{x_E^*}\alpha_E^*\left[\pi-\frac{\alpha_E^*\sigma^2}{2}\right]d\sigma^2+\int_{x_E^*}^{1+\Delta}\alpha_E^A(\sigma^2)\left[\pi-\frac{\alpha_E^A(\sigma^2)\sigma^2}{2}\right]d\sigma^2\right]=$$
$$\frac{\gamma\pi^2}{2\Delta}\left[\lambda+\frac{(1+\lambda)(1-\lambda)}{2}\ln\frac{(1+\Delta)\,(1-\lambda)}{(1-\Delta)\,(1+\lambda)}\right].$$

Next, consider the case of non-exclusive advice. Two cases must be distinguished depending on the type of equilibrium.

(1) In the region of parameters where there exist an equilibrium with partial delegation where both investors pool for low values of σ^2 and delegate otherwise — i.e., see Proposition 2 and the first part of Proposition 3 — the investors' expected utility is

$$\mathcal{W}_N^*=\frac{\gamma}{2\Delta}\left[\int_{1-\Delta}^{x_N^*}\alpha_N^*\left[\pi-\frac{\alpha_N^*\sigma^2}{2}\right]d\sigma^2+\int_{x_N^*}^{1+\Delta}\alpha_N^A(\sigma^2)\left[\pi-\frac{\alpha_N^A(\sigma^2)\sigma^2}{2}\right]d\sigma^2\right]=$$
$$\frac{\gamma\pi^2}{2\,(1+\theta)\,\Delta}\left[\lambda-\theta+(1+\lambda)\,\frac{1-\lambda+2\theta}{2\,(1+\theta)}\ln\frac{(1+\Delta)\,(1-\lambda+2\theta)}{(1-\Delta)\,(1+\lambda)}\right]$$

Notice that in this region of parameters $\mathcal{W}_E^*=\mathcal{W}_N^*$ for $\theta=0$ and that

$$\frac{\partial\mathcal{W}_N^*}{\partial\theta}=\frac{\gamma\pi^2}{2\Delta}\frac{(1+\lambda)}{(1+\theta)^3}\left[(\lambda-\theta)\ln\frac{(1+\Delta)\,(1-\lambda+2\theta)}{(1-\Delta)\,(1+\lambda)}\right]>0$$

since $\lambda>\theta$ and $\frac{(1+\Delta)(1-\lambda+2\theta)}{(1-\Delta)(1+\lambda)}>1$. It then follows that $\mathcal{W}_E^*\geq\mathcal{W}_N^*$ if and only if $\theta\leq 0$.

(2) In the region of parameters where there exist an equilibrium with partial delegation where both investors pool for high values of σ^2 and delegate otherwise — i.e., see the second part of Proposition 3 — the investors' expected utility is

$$\mathcal{W}_N^*=\frac{\gamma}{2\Delta}\left[\int_{1-\Delta}^{x_N^*}\alpha_N^A(\sigma^2)\left[\pi-\frac{\alpha_N^A(\sigma^2)\sigma^2}{2}\right]d\sigma^2+\int_{x_N^*}^{1+\Delta}\alpha_N^*\left[\pi-\frac{\alpha_N^*\sigma^2}{2}\right]d\sigma^2\right]=$$
$$\frac{\gamma\pi^2}{2\,(1+\theta)\,\Delta}\left[\theta-\lambda+(1+\lambda)\,\frac{1-\lambda+2\theta}{2\,(1+\theta)}\ln\frac{(1+\Delta)\,(1+\lambda)}{(1-\Delta)\,(1-\lambda+2\theta)}\right].$$

Notice that $\mathcal{W}_E^* = \mathcal{W}_N^*$ for $\theta = \frac{2\lambda}{1-\lambda} > \lambda$. Moreover,

$$\frac{\partial \mathcal{W}_N^*}{\partial \theta} = \frac{\gamma\,(\lambda-\theta)\,(1+\lambda)}{(1+\theta)^3} \ln \frac{(1+\Delta)\,(1+\lambda)}{(1-\Delta)\,(1-\lambda+2\theta)} < 0,$$

since $\lambda < \theta$ and

$$\frac{1+\lambda}{1-\lambda+2\theta}\frac{1+\Delta}{1-\Delta} > 1.$$

Hence, $\mathcal{W}_E^* \geq \mathcal{W}_N^*$ if and only if $\theta \geq \frac{2\lambda}{1-\lambda}$. ■

Proof of Proposition 5. In the case of exclusivity, the average investment into the risky asset is

$$\hat{\alpha}_E^* = \frac{1}{2\Delta}\left[\int_{1-\Delta}^{x_E^*} \alpha_E^* d\sigma^2 + \int_{x_E^*}^{1+\Delta} \alpha_E^A(\sigma^2) d\sigma^2\right] = \frac{\pi}{\Delta}\left[\lambda + \frac{(1+\lambda)}{2}\ln\frac{(1+\Delta)\,(1-\lambda)}{(1-\Delta)\,(1+\lambda)}\right].$$

By contrast, in the region of parameters in which there exists an equilibrium with partial delegation where both investors pool for low values of σ^2 and delegate otherwise — i.e., see Proposition 2 and the first part of Proposition 3 — the average investment into the risky asset is

$$\hat{\alpha}_N^* = \frac{1}{2\Delta}\left[\int_{1-\Delta}^{x_N^*} \alpha_N^* d\sigma^2 + \int_{x_N^*}^{1+\Delta} \alpha_N^A(\sigma^2) d\sigma^2\right] = \frac{\pi}{\Delta(1+\theta)}\left[\lambda - \theta + \frac{1+\lambda}{2}\ln\frac{(1+\Delta)\,(1-\lambda+2\theta)}{(1-\Delta)\,(1+\lambda)}\right].$$

Hence

$$\hat{\alpha}_E^* \geq \hat{\alpha}_N^* \quad \Leftrightarrow \quad 2\theta - \ln\left[\frac{1-\lambda+2\theta}{1-\lambda}\left(\frac{(1-\Delta)\,(1+\lambda)}{(1+\Delta)\,(1-\lambda)}\right)^{\theta}\right] \geq 0.$$

The solution of $\hat{\alpha}_N^* = \hat{\alpha}_E^*$ with respect to Δ is

$$\Delta_1 = \frac{\frac{1+\lambda}{1-\lambda}\left(\frac{1-\lambda+2\theta}{(1-\lambda)\exp 2\theta}\right)^{\frac{1}{\theta}} - 1}{\frac{1+\lambda}{1-\lambda}\left(\frac{1-\lambda+2\theta}{(1-\lambda)\exp 2\theta}\right)^{\frac{1}{\theta}} + 1},$$

where it can be checked that in the parameter region under consideration $\Delta_1 \in (0,1)$. Notice also that

$$\frac{\partial \ln\left[\frac{1-\lambda+2\theta}{1-\lambda}\left(\frac{(1-\Delta)(1+\lambda)}{(1+\Delta)(1-\lambda)}\right)^{\theta}\right]}{\partial \Delta} \geq 0 \quad \Leftrightarrow \quad \theta \leq 0,$$

which directly implies the result.

Consider now the region of parameters where there exists an equilibrium with partial delegation where both investors pool for high values of σ^2 and delegate otherwise — i.e., see the second part of Proposition 3 — the average investment into the risky asset is

$$\hat{\alpha}_N^* \equiv \frac{1}{2\Delta}\left[\int_{1-\Delta}^{x_N^*} \alpha_N^A(\sigma^2) d\sigma^2 + \int_{x_N^*}^{1+\Delta} \alpha_N^* d\sigma^2\right] = \frac{\pi}{\Delta\,(1+\theta)}\left[\theta - \lambda + \frac{1+\lambda}{2}\ln\frac{(1+\Delta)\,(1+\lambda)}{(1-\Delta)\,(1-\lambda+2\theta)}\right].$$

Hence

$$\hat{\alpha}_E^* \geq \hat{\alpha}_N^* \quad \Leftrightarrow \quad 2\left(2\lambda - \theta\left(1-\lambda\right)\right) - \left(1+\lambda\right)\ln\left[\frac{\left(1+\lambda\right)^{(2+\theta)}}{\left(1-\lambda+2\theta\right)\left(1-\lambda\right)^{\theta}}\left(\frac{1-\Delta}{1+\Delta}\right)^{\theta}\right] > 0.$$

The solution of $\hat{\alpha}_N^* = \hat{\alpha}_E^*$ with respect to Δ is

$$\Delta_2 = \frac{\left[\frac{(1-\lambda+2\theta)(1-\lambda)^{\theta}}{(1+\lambda)^{(2+\theta)}}\exp\left(\frac{2(2\lambda-\theta+\lambda\theta)}{1+\lambda}\right)\right]^{-\frac{1}{\theta}} - 1}{\left[\frac{(1-\lambda+2\theta)(1-\lambda)^{\theta}}{(1+\lambda)^{(2+\theta)}}\exp\left(\frac{2(2\lambda-\theta+\lambda\theta)}{1+\lambda}\right)\right]^{-\frac{1}{\theta}} + 1} < 1$$

Suppose that $\Delta_2 > 0$. Then,

$$\frac{\partial \ln\left[\frac{(1+\lambda)^{(2+\theta)}}{(1-\lambda+2\theta)(1-\lambda)^{\theta}}\left(\frac{1-\Delta}{1+\Delta}\right)^{\theta}\right]}{\partial\Delta} < 0,$$

since $\theta > 0$ in the parameter region under consideration. This implies that $\alpha_E^* \geq \alpha_N^*$ if and only if $\Delta \geq \Delta_2$. We now show under which conditions $\Delta_2 > 0$. Notice that this requires

$$\frac{\left(1-\lambda+2\theta\right)\left(1-\lambda\right)^{\theta}}{\left(1+\lambda\right)^{(2+\theta)}}\exp\left(\frac{2\left(2\lambda-\theta+\lambda\theta\right)}{1+\lambda}\right) < 1,$$

which implies

$$\Phi\left(\lambda,\theta\right) \equiv \ln\frac{\left(1-\lambda+2\theta\right)\left(1-\lambda\right)^{\theta}}{\left(1+\lambda\right)^{(2+\theta)}} + \frac{2\left(2\lambda-\theta+\lambda\theta\right)}{1+\lambda} < 0.$$

Notice that $\Phi\left(\lambda=0,\theta\right) = -2\theta+\ln\left(2\theta+1\right) < 0$ for each $\theta \in [0,1]$ and $\Phi\left(\lambda=\theta,\theta\right) = \ln\frac{(1-\theta)^{\theta}}{(1+\theta)^{(1+\theta)}} + 2\theta < 0$ if and only if $\theta > 0.537$. It is then easy to show that there exists a threshold $\lambda^* < \theta$ such that: (*i*) if $\lambda > \lambda^*$ then $\Phi\left(\lambda,\theta\right) < 0$ for every θ; (*ii*) if $\lambda \leq \lambda^*$ there exists a function $\theta\left(\lambda\right)$, which solves $\Phi\left(\lambda,\theta\right) = 0$, such that $\Phi\left(\lambda,\theta\right) < 0$ if and only if $\theta > \theta\left(\lambda\right)$. Hence, $\Delta_2 > 0$ if: (*i*) $\lambda \geq \lambda^*$; (*ii*) $\lambda < \lambda^*$and $\theta > \theta\left(\lambda\right)$. ■

Proof of Proposition 6. The proof of this result hinges on the same techniques used in the proof of Propositions 2-3. So it will be omitted. ■

Proof of Proposition 7. First, showing that for $\theta < 0$ investor 2's optimal asset allocation has the same features as that of Proposition 2 is straightforward: in this case $\alpha_2^D(\sigma^2) > \alpha^F(\sigma^2)$ for every σ^2, which means that investor's 2 optimal mechanism requires a cap on the amount of wealth invested into the risky asset.

Hence, in the rest of the proof we will only consider $\theta > 0$. Assume that investor 2 's optimal allocation rules is such that

$$\tilde{\alpha}(\sigma^2) = \begin{cases} \overline{\alpha}^P & \Leftrightarrow \quad \sigma^2 < \underline{\sigma}_2^2 \\ \alpha^P\left(\sigma^2\right) & \Leftrightarrow \quad \sigma^2 \in \left[\underline{\sigma}_2^2, \overline{\sigma}_2^2\right] \\ \underline{\alpha}^P & \Leftrightarrow \quad \sigma^2 > \overline{\sigma}_2^2 \end{cases}, \tag{.18}$$

with $1+\Delta > \overline{\sigma}_2^2 > \underline{\sigma}_2^2 > 1-\Delta$. First, note that by the incentive compatibility constraint 6.2 it follows immediately that investor 2's optimal asset allocation $\tilde{\alpha}(\sigma^2)$ is non-increasing in σ^2. Hence, $\overline{\alpha}^P > \alpha^P\left(\sigma^2\right) > \underline{\alpha}^P$. Investor 2's expected utility is

$$\int_{1-\Delta}^{\underline{\sigma}^2} \overline{\alpha}^P\left[\pi - \frac{\overline{\alpha}^P\sigma^2}{2}\right]d\sigma^2 + \int_{\underline{\sigma}^2}^{\overline{\sigma}^2} \alpha_2^A(\sigma^2)\left[\pi - \frac{\alpha_2^A(\sigma^2)\sigma^2}{2}\right]d\sigma^2 + \int_{\overline{\sigma}^2}^{1+\Delta} \underline{\alpha}^P\left[\pi - \frac{\underline{\alpha}^P\sigma^2}{2}\right]d\sigma^2,$$

Maximizing this function with respect to $\overline{\alpha}^P$ and $\underline{\alpha}^P$

$$\overline{\alpha}^P = \frac{\pi}{\mathbb{E}[\sigma^2|\sigma^2 \le \underline{\sigma}_2^2]}, \qquad \underline{\alpha}^P = \frac{\pi}{\mathbb{E}[\sigma^2|\sigma^2 \ge \overline{\sigma}_2^2]}.$$

Moreover, maximizing with respect to $\underline{\sigma}^2$ and $\overline{\sigma}^2$

$$(1+\lambda)\,\alpha^F(\underline{\sigma}^2) - \theta = \frac{\pi}{\mathbb{E}[\sigma^2|\sigma^2 \le \underline{\sigma}^2]}, \qquad (1+\lambda)\,\alpha^F(\overline{\sigma}^2) - \theta = \frac{\pi}{\mathbb{E}[\sigma^2|\sigma^2 \ge \overline{\sigma}^2]},$$

which can be rewritten as

$$\theta(\underline{\sigma}^2) \equiv \frac{1+\lambda}{\underline{\sigma}^2} - \frac{2}{1-\Delta+\underline{\sigma}^2} = \frac{\theta}{\pi}, \tag{.19}$$

$$\Phi(\overline{\sigma}^2) \equiv \frac{1+\lambda}{\overline{\sigma}^2} - \frac{2}{1+\Delta+\overline{\sigma}^2} = \frac{\theta}{\pi}. \tag{.20}$$

Notice that $\Phi\left(\sigma^2\right) > \theta\left(\sigma^2\right)$ for every σ^2. Moreover,

$$\Phi\left(1-\Delta\right) = \frac{\lambda+\Delta}{1-\Delta} > \Phi(1+\Delta) = \frac{\lambda}{1+\Delta}, \qquad \theta\left(1-\Delta\right) = \frac{\lambda}{1-\Delta} > \theta(1+\Delta) = \frac{\lambda-\Delta}{1+\Delta}.$$

It is easy to verify that

$$\Phi\left(1-\Delta\right) > \theta\left(1-\Delta\right) > \Phi\left(1+\Delta\right) > \theta\left(1-\Delta\right),$$

which implies that if there exists a solution to (.19)-(.20), it must be the case that $\overline{\sigma}^2 > \underline{\sigma}^2$.

Suppose first that $\lambda > \Delta$, so that $\theta\left(1-\Delta\right) > 0$. A necessary and sufficient condition for the asset allocation rule described in .18 to be optimal is

$$\theta\left(1-\Delta\right) > \frac{\theta}{\pi} > \Phi\left(1+\Delta\right) \quad \Leftrightarrow \quad \gamma \in \left(\frac{\Phi\left(1+\Delta\right)\left(\mu-r_f\right)}{\theta}, \frac{\theta\left(1-\Delta\right)\left(\mu-r_f\right)}{\theta}\right).$$

Setting $\underline{\gamma} = \Phi\left(1+\Delta\right)\left(\mu-r_f\right)/\theta$ and $\overline{\gamma} = \theta\left(1-\Delta\right)\left(\mu-r_f\right)/\theta$ completes the first part of the proof.

Next, using the same type of technique, it is easy to show that investor 2's optimal asset allocation rule has the same features as in Proposition 2 for $\gamma \le \underline{\gamma}$ — i.e., it requires to cap the investment into the risky asset — while it has the same features as in the second part of Proposition 3 if $\gamma \ge \overline{\gamma}$ — i.e., it requires investor 2's to impose a floor on the amount of wealth allocated to the risky asset. ∎

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