# Cooperation vs. Collusion: How Essentiality Shapes Co-opetition*

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October 23, 2013

**Abstract**

The paper makes two related contributions. First, and in contrast with the rich body of literature on collusion with (mainly perfect) substitutes, it derives general results on the sustainability of tacit coordination for a class of nested demand functions that allows for the full range between perfect substitutes and perfect complements.

Second, it studies the desirability of joint marketing alliances, an alternative to mergers. It shows that a combination of two information-free regulatory requirements, mandated unbundling by the joint marketing entity and unfettered independent marketing by the firms, makes joint-marketing alliances always socially desirable, whether tacit coordination is feasible or not.

*Keywords*: tacit collusion, cooperation, substitutes and complements, essentiality, joint marketing agreements, patent pools, independent licensing, unbundling, co-opetition.

*JEL numbers*: D43, L24, L41, O34.

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*The research leading to these results has received funding from the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013) Grant Agreement no. 249429 and from the National Science Foundation (NSF grant "Patent Pools and Biomedical Innovation", award #0830288). The authors are members of IDEI, whose IP research program is funded by Microsoft, Orange and Qualcomm. They are grateful to Aleksandra Boutin, Georgy Egorov, Michael Katz, Volker Nocke, Carl Shapiro, Glen Weyl, and participants at the 12th CSIO-IDEI conference, the IO Workshop of the 2013 NBER Summer Institute, the 8th CRESSE conference, seminars in Edinburgh and Mannheim, and the sixth Antitrust Economics and Competition Policy conference (Northwestern), for helpful comments.

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# 1 Introduction

## 1.1 Paper's contribution

The assessment of public policies regarding oligopolies (structural remedies and merger analysis, regulation of transparency and other facilitating practices, treatment of joint marketing alliances such as patent pools...) requires forming an opinion on whether such policies are likely to hinder or facilitate tacit collusion. Yet, products rarely satisfy the axiom of substitutability that underlies our rich body of knowledge on the topic. For instance, competitors in a technological class not only exhibit various forms of differentiation, but often they are also complementors: network externalities facilitate the adoption of their technology and deter the emergence of rivals using alternative approaches. This paper's first contribution is to provide a study of collusion for a class of demand functions allowing for the full range between perfect substitutes and perfect complements.

To achieve this, we adopt a nested demand model in which individual users must select a) which products to purchase in the technological class and b) whether to adopt the technology at all. The first choice depends on the extent of product substitutability within the class, while the second captures the complementarity dimension. We measure the "essentiality" of offerings through the reduction in the user's value of the technology when he foregoes an offering. For the sake of tractability, users have the same preferences along this dimension, and only differ along another dimension: the cost of adopting the technology, or equivalently their opportunity cost of not adopting another technology. Within this framework, we derive general results about the sustainability of "tacit collusion" (coordinated increase in price) or "tacit cooperation" (coordinated decrease in price), that is, about bad and good coordination through repeated interaction.

When essentiality is low, firms are rivals and would like to raise price; yet, and except in the extreme case of perfect substitutes, such tacit collusion leads users to forego part of the technology, as the prices of the components do not vindicate acquiring all. This additional distortion, beyond the standard suboptimal quantity, both acts as a partial deterrent to collusion and makes the latter, if it happens, socially even more costly. This shows that two instruments that we usually treat as identical, merger and collusion, in fact differ whenever users are not limited to buying a single product; the gap stems from the ability to bundle, which is not available under mere collusion. We show that collusion is feasible when firms are patient and essentiality is low.

Beyond some essentiality threshold, firms become complementors and would like to lower price toward the joint-profit-maximizing price. Such tacit cooperation is feasible provided that the firms are patient enough; it is also easier to enforce, the higher the essentiality parameter. Overall, coordination is most likely, the most inefficient the non-coordinated equilibrium.

The paper's second contribution is the analysis of tacit collusion under joint and independent marketing. It is well-known that when products exhibit complementarities, joint marketing alliances ("JMAs," hereafter) have the potential of preventing multiple marginalization. Yet authorities are never quite sure whether products are complements or substitutes; such knowledge requires knowing the demand function and the field of use; the pattern of complementarity/substitutability may also vary over time. Lerner and Tirole (2004) however showed that, in the absence of tacit coordination, joint marketing is always socially desirable if firms keep ownership of, and thereby are able to independently market their offering. A nice property of this "perfect screen" is that it requires no information about essentiality when considering whether to allow JMAs. And indeed, in practice antitrust agencies are more lenient toward JMAs that let firms freely market their products outside the JMA.

To see whether the "perfect screen" result of Lerner and Tirole extends to the possibility of tacit coordination, we characterize optimal tacit coordination when firms are allowed to form a pool under the independent licensing provision. The pool enables the firms to lower price when firms are complementors. It also prevents the collusion inefficiency stemming from selling an incomplete technology at a high quality-adjusted price when firms are strong substitutes. However, the pool may also facilitate collusion. By eliminating the inefficiency from selling an incomplete technology (the corollary of an attempt to raise price in the absence of a pool), the pool makes high prices more attractive to the firms. Thus, authorities run the risk of approving a welfare-decreasing JMA among producers of weak substitutes.

Tacit coordination thus poses a new challenge: Independent licensing no longer is a perfect screen. We show that another information-free instrument, the "unbundling requirement" that the JMA market individual pieces at a total price not exceeding the bundle price, can be appended so as to re-create a perfect screen, and that both instruments are needed to achieve this. In essence, this requires JMAs to set *price caps* for all licenses and to refrain from offering bundle discounts. JMAs that set price caps and allow individual licensing are always welfare-increasing.

The paper is organized as follows. In the remainder of this section, we first provide further motivation in the context of patents held by different companies, and then explain why our framework applies way beyond the intellectual property realm; and we relate our contribution to the existing literature. For expositional ease, we then derive our main results for the symmetric duopoly case. Section 2 develops the nested-demand framework in the absence of joint marketing and derives the uncoordinated equilibrium: As essentiality increases, firms are first rivals, then weak complementors and finally strong complementors. Section 3 studies tacit coordination in this framework. Section 4 introduces joint marketing, subject to firms keeping ownership of their product, and analyses whether this institution has the potential to raise or lower price. Section 5 derives the information-free regulatory requirement. Section 6 shows that our insights carry over when firms' R&D intensity is endogenous. Section 7 demonstrates the results' robustness to asymmetric essentiality and to an arbitrary number of products. Section 8 concludes. Most proofs are sketched in the text and developed in more detail in the Appendices.

## 1.2 Illustration: the market for intellectual property

In industries such as software and biotech, the recent inflation in the number of patents has led to a serious concern about users' ability to build on the technology without infringing on intellectual property (IP hereafter). The patent thicket substantially increases the transaction costs of assembling licenses and raises the possibility of royalty stacking or unwanted litigation. To address these problems, academics, antitrust practitioners and policy-makers have proposed that IP owners be able to bundle and market their patents within patent pools. And indeed, since the first review letters of the US Department of Justice in the 90s and similar policies in Europe and Asia, patent pools are enjoying a revival.[1]

Patent pools however are under sharp antitrust scrutiny as they have the potential to enable the analogue of "mergers for monopoly" in the IP domain. Focusing on the two polar cases, patent pools are socially detrimental in the case of perfectly substitutable patents (they eliminate Bertrand competition) and beneficial for perfectly complementary patents (they prevent Cournot $n^{th}$ marginalization). More generally, they are more likely to raise welfare, the more complementary the patents involved in the technology. But in this grey zone, antitrust authorities have little information as to the degree of complementarity, which furthermore changes over time with the emergence of new products and technologies. Demand data are rarely available and, to make matters worse, patents can be substitute at some prices and complements at others. Thus patent pool regulation occurs under highly incomplete information. Yet a covenant requiring no information- specifying that patent owners keep property of their patent, so that the pool only performs common marketing- can perfectly screen in welfare-enhancing pools and out welfare-reducing pools; this result, due to Lerner and Tirole (2004), holds even if patents have asymmetric importance. Interestingly, this "independent licensing" covenant has been required lately by antitrust authorities in the US, Europe, and Japan for instance.

[1] Before WWII, most of the high-tech industries of the time were organized around patent pools; patent pools almost disappeared in the aftermath of adverse decisions by the US Supreme Court.

Because of the simplicity of this screening device and the importance of patent pools for the future of innovation and its diffusion, its efficacy should be explored further. Indeed, nothing is known about its properties in a repeated-interaction context (the literature so far has focused on static competition). Independent licensing enables deviations from a collusive pool price when patents are sufficiently substitutable so as to make the pool welfare-reducing; but it may also facilitate the punishment of deviators.

## 1.3 A broader applicability

While we motivate our analysis through the intellectual property lens, the analysis and the general insights that products can be both complements and substitutes, and that information-free screens exist that make such JMSs desirable, have much broader applicability, as illustrated by the following examples.[2]

Cable and satellite television operators offer bundles of contents. These contents compete among themselves for viewer attention; but they are also complements to the extent that increased TV operator membership, which in particular hinges on the individual fees they charge, benefits all content providers.

Payment systems using a common point-of-sale terminal or interface at merchant premises face a similar situation: they compete for cardholder clientele and usage. At the same time, they share a common interest in merchants' wide adoption of the terminals.

Health care providers who are members of a health insurance network vie for patients insured by the network but also depend on rival providers for the attractiveness of the insurance network (Katz 2011). Thus hospitals can be both substitutes (because they compete with one another to attract insured patients) and complements (because they jointly determine the value of becoming insured by the network).

More generally, the second part of this paper, concerned with JMAs, analyses the conduct and performance of industries with product portfo-

[2] We are grateful to Carl Shapiro and Michael Katz for providing some of these examples.

lios, where these portfolios may assemble TV channels, payment systems, hospitals, or to add two further illustrations, music performance rights (as, say, licensed by Pandora), alcoholic beverages (as in the GrandMet-Guinness merger), retail outlets (in department stores and commercial malls), or books, tickets and hotel rooms (on online platforms).

## 1.4 Relation to the literature

There is no point reviewing here the rich literature on repeated interactions with and without observability of actions. By contrast, applications to non-homogeneous oligopolies are scarcer, despite the fact that antitrust authorities routinely consider the possibility of tacit collusion in their merger or marketing alliances decisions.

The exception to this overall neglect is a literature which, following Deneckere (1983) and Wernerfelt (1989), studies the impact of product differentiation. The conventional view, pioneered by Stigler (1964), is that homogeneous cartels are more stable than non-homogeneous ones (Jéhiel (1992) calls this the principle of minimum differentiation). In the context of symmetric horizontal differentiation, Ross (1992) shows however that stability does not increase monotonically with substitutability, because product differentiation both lowers the payoff from deviation and reduces the severity of punishments (if one restricts attention to Nash reversals; Häckner (1996) shows that Abreu's penal codes can be used to provide more discipline than Nash reversals, and finds that product differentiation facilitates collusion).[3] Building on these insights, Lambertini *et al.* (2002) argue that, by reducing product variety, joint ventures can actually destabilize collusion.

In a context of vertical differentiation, where increased product diversity also implies greater asymmetry among firms, Häckner (1994) finds that collusion is instead easier to sustain when goods are more similar (and thus firms are more symmetric). Building on this insight, Ecchia and Lambertini (1997) note that introducing or raising a quality standard can make collusion less

[3] Raith (1996) emphasizes another feature of product differentiation, namely, the reduced market transparency that tends to hinder collusion.

sustainable.

This paper departs from the existing literature in several ways. First, it studies collusion with (varying degrees of) complementarity and not only substitutability. It characterizes optimal tacit coordination when products range from perfect substitutes to perfect complements.[4] Second, it allows for JMAs and studies whether tacit collusion undermines these alliances. Finally, it derives regulatory implications.

# 2 The model

## 2.1 Framework

For expositional purposes,we develop the model using the language of intellectual assets and licensing instead of goods and sales; but the model applies more broadly to repeated interactions within arbitrary industries. We assume that the technology is covered by patents owned by separate firms (two in the basic version). To allow for the full range between perfect substitutes and perfect complements while preserving tractability, we adopt a nested demand model in which the individual users must select a) which patents to acquire access to if they adopt the technology and b) whether they adopt the technology at all.

Users differ in one dimension: the cost of adopting the technology or, equivalently, their opportunity cost of adopting another technology. There are thus two elasticities in this model: the intra-technology elasticity which reflects the ability/inability of users to opt for an incomplete set of licenses; and the inter-technology elasticity. The simplification afforded by this nested model is that, conditionally on adopting the technology, users have identical preferences over IP bundles. This implies that under separate marketing, technology adopters all select the same set of licenses; furthermore, a JMA

[4] Deneckere (1983) actually considers complements as well as substitutes (for linear demand and Nash-reversal punishments). His setting however does not cover the product portfolio paradigm considered here. Despite Deneckere's early exploration of a broad spectrum of substitutability/complementarity, the subsequent literature has focused on (imperfect) substitutes.

need not bother with menus of offers (second-degree price discrimination).

There are two firms, $i = 1, 2$, and a mass 1 of users. Each firm owns a patent pertaining to the technology. While users can implement the technology by building on a single patent, it is more effective to combine both: users obtain a gross benefit $V$ from the two patents, and only $V - e$ with either patent alone. The parameter $e \in [0, V]$ measures the *essentiality* of individual patents: these are clearly not essential when $e$ is low (in the limit case $e = 0$, the two patents are perfect substitutes), and become increasingly essential as $e$ increases (in the limit case $e = V$, the patents are perfect complements, as each one is needed in order to develop the technology). The extent of essentiality is assumed to be known by IP owners and users; for policy purposes, it is advisable to assume that policymakers have little knowledge of the degree of essentiality.

Adopting the technology involves an opportunity cost, $\theta$, which varies across users and has full support $[0, V]$ and c.d.f $F(\theta)$. A user with cost $\theta$ adopts the technology if and only if $V \geq \theta + P$, where $P$ is the total licensing price. The demand for the bundle of the two patents licensed at price $P$ is thus

$$D(P) \equiv F(V - P).$$

Similarly, the demand for a single license priced $p$ is

$$D(p + e) = F(V - e - p).$$

That is, an incomplete technology sold at price $p$ generates the same demand as the complete technology sold at price $p + e$; thus $p + e$ will be labelled the "quality-adjusted price."

Users obtain a net surplus $S(P)$ when they buy the complete technology at total price $P$, where $S(P) \equiv \int_0^{V-P} (V - P - \theta)\, dF(\theta) = \int_P^V D(\tilde{P}) d\tilde{P}$, and a net surplus $S(p + e)$ from buying an incomplete technology at price $p$.

To ensure the concavity of the relevant profit functions, we will assume that the demand function is well-behaved:

**Assumption A**: $D(.)$ is twice continuously differentiable and, for any $P \geq 0$, $D'(P) < 0$ and $D'(P) + PD''(P) < 0$.

If users buy the two licenses at unit price $p$, each firm obtains

$$\pi(p) \equiv pD(2p),$$

which is strictly concave under Assumption A; let $p^m \in [0, V]$ denote the per-patent monopoly price:

$$p^m \equiv \arg\max_p \{\pi(p)\}.$$

If instead users buy a single license at price $p$, industry profit is

$$\tilde{\pi}(p) \equiv pD(p+e),$$

which is also strictly concave under Assumption A; let $\tilde{p}^m(e)$ denote the monopoly price for an incomplete technology:

$$\tilde{p}^m(e) \equiv \arg\max_p \{pD(p+e)\}.$$

Finally, let

$$\pi^m \equiv \pi(p^m) = p^m D(2p^m)$$

and

$$\tilde{\pi}^m(e) \equiv \tilde{\pi}(\tilde{p}^m(e)) = \tilde{p}^m(e) D(\tilde{p}^m(e) + e)$$

denote the highest possible profit per *licensing* firm when two or one patents are licensed. $\tilde{\pi}^m(e)$ is decreasing in $e$.

## 2.2 Static non-cooperative pricing

Consider the static game in which the two firms simultaneously set their prices. Without loss of generality we require prices to belong to the interval $[0, V]$. When a firm raises its price, either of two things can happen: First, the technology adopters may stop including the license in their basket; second, they may keep including the license in their basket, but because the technology has become more expensive, fewer users adopt it.

Let us start with the latter case. In reaction to price $p_j$ set by firm $j$, firm $i$ sets price $r(p_j)$ given by:

$$r(p_j) \equiv \arg\max_{p_i}\{p_i D(p_i + p_j)\}.$$

Assumption A implies price absorption:[5]

$$-1 < r'(p_j) < 0.$$

The two patents are then both complements (the demand for one decreases when the price of the other increases) and strategic substitutes (an increase in the price of the other patent induces the firm to lower its own price). Furthermore, $r'(.) > -1$ implies that $r(.)$ has a unique fixed point, which we denote by $\hat{p}$:

$$\hat{p} = r(\hat{p}).$$

Double marginalization holds:[6] $\hat{p} > p^m$.

Firms' ability to raise price without being dropped from the users' basket requires that, for all $i$, $p_i \leq e$. Firm $i$'s best response to firm $j$ setting price $p_j \leq e$ is to set

$$p_i = \min\{e, r(p_j)\}.$$

When instead $p_j > e$, then firm $i$ faces no demand if $p_i > p_j$ (as users buy only the lower-priced license), and faces demand $D(p_i + e)$ if $p_i < p_j$. Competition then drives prices down to $p_1 = p_2 = e$. It follows that the Nash equilibrium is unique and symmetric: Both firms charge

$$p^N \equiv \min\{e, \hat{p}\},$$

and face positive demand. Figure 1 summarizes the static analysis.

We will denote the resulting profit by

$$\pi^N \equiv \pi\left(p^N\right).$$

[5] See Online Appendix A for a detailed analysis of the properties of reaction functions.

[6] By revealed preference, $p^m D(2p^m) \geq \hat{p}D(2\hat{p}) \geq p^m D(\hat{p} + p^m)$ and thus $D(2p^m) \geq D(\hat{p}+p^m)$ implying $\hat{p} \geq p^m$. Assumption $A$ moreover implies that this inequality is strict.

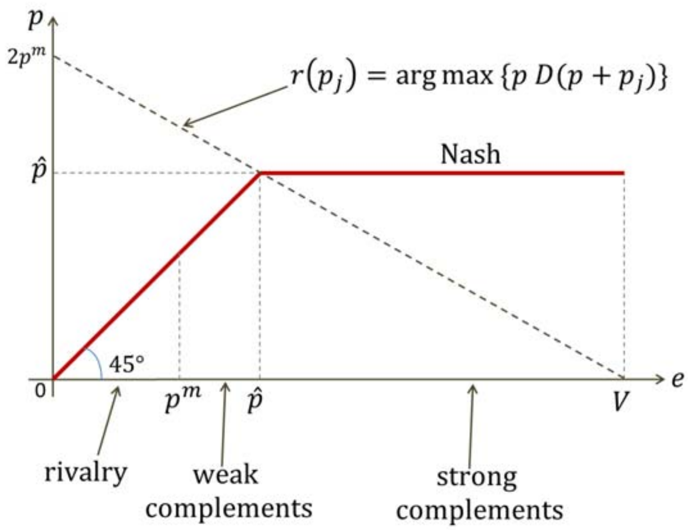


Figure 1: no coordinated effects

In what follows, we will vary $e$ and keep $V$ constant; keeping the technology's value $V$ constant keeps invariant the reaction function $r(.)$ and its fixed point $\hat{p}$, as well as the optimal price and profit, $p^m$ and $\pi^m$, which all depend only on $V$. By contrast, $\tilde{p}^m(e)$ and $\tilde{\pi}^m(e)$, and possibly the Nash price and profit, $p^N$ and $\pi^N$, vary with $e$.

# 3 Tacit coordination

We now suppose that the firms play the game repeatedly, with discount factor $\delta \in (0,1)$, and we look for the best (firm-optimal) pure-strategy tacit coordination equilibria. Let $v_i = (1-\delta)\Sigma_{t\geq 0}\delta^t \pi_i^t$ denote firm $i$'s average discounted profit over the entire equilibrium path, and

$$\bar{v} \equiv \max_{(v_1,v_2)\in\mathcal{E}} \left\{ \frac{v_1+v_2}{2} \right\}$$

denote the maximal per firm equilibrium payoff in the set $\mathcal{E}$ of pure-strategy equilibrium payoffs.[7] Tacit coordination raises profits only if $\bar{v} > \pi^N$.

The location of $e$ with respect to $p^m$ drives the nature of tacit coordination:

• If $e < p^m$ (which implies $e < \hat{p}$ and thus $p^N = e$) the firms will seek to *raise* price above the static Nash level; we will refer to such tacit coordination as *collusion*, as it reduces social welfare. Note that charging a price above $p^N = e$ induces users to buy at most one license. We will assume that firms can share the resulting profit $\tilde{\pi}(p)$ as they wish: in our setting, they can do so by charging the same price $p > e$ and allocating market shares among them; more generally, introducing a small amount of heterogeneity in users' preferences would allow the firms to achieve arbitrary market shares by choosing their prices appropriately. In this incomplete-technology region, it is optimal for the firms to raise the price up to $\tilde{p}^m(e)$, if feasible, and share the resulting profit, $\tilde{\pi}^m(e)$.

• If $e > p^m$, the firms will seek to *lower* price below the static Nash level; we will refer to such tacit coordination as *cooperation*, as it benefits users as well as firms. Ideally, the firms would reduce the per-patent price down to $p^m$, and share profit $\pi^m$.

Likewise, the location of $e$ with respect to $\hat{p}$ conditions minmax profits:

**Lemma 1 (minmax)** *Let $\underline{\pi}$ denote the minmax payoff.*

*i) If $e \leq \hat{p}$, the static Nash equilibrium $(e, e)$ gives each firm the minmax profit: $\underline{\pi} = \pi^N = \pi(e)$.*

*ii) If $e > \hat{p}$, the minmax profit is the incomplete-technology per-period monopoly profit: $\underline{\pi} = \widetilde{\pi}^m(e) < \pi^N = \pi(\hat{p})$.*

[7] This maximum is well defined, as the set $\mathcal{E}$ of subgame perfect equilibrium payoffs is compact; see, e.g., Mailath and Samuelson (2006), chapter 2. Also, although we restrict attention to pure-strategy subgame perfect equilibria here, the analysis could be extended to public mixed strategies (where players condition their strategies on public signals) or, in the case of private mixed strategies, to perfect public equilibria (relying on strategies that do not condition future actions on private past history); see Mailath and Samuelson (2006), chapter 7.

**Proof.** To establish part *i)*, note that firm $i$ can secure its presence in the users' basket by charging $e$, thus obtaining $eD(e+p_j)$ if $p_j \le e$ and $eD(2e)$ if $p_j > e$. Either way it can secure at least $\pi(e) = eD(2e)$. Because for $e \le \hat{p}$ this lower bound is equal to the static Nash profit, we have $\underline{\pi} = \pi^N = \pi(e)$.

We now turn to part *ii)*. If firm $j$ sets a price $p_j \ge e$, firm $i$ can obtain at most $\max_{p \le p_j} pD(e+p) = \tilde{\pi}^m(e)$ (as $\tilde{p}^m(e) = r(e) < \hat{p} < e \le p_j$). Setting instead a price $p_j < e$ allows firm $i$ to obtain at least $\max_{p \le e} pD(p_j + p) > \max_{p \le e} pD(e+p) = \tilde{\pi}^m(e)$. Therefore, setting any price above $e$ minmaxes firm $i$, which then obtains $\tilde{\pi}^m(e)$. ■

Thus, the location of $e$ with respect to $\hat{p}$ affects the scope for punishments. When $e \le \hat{p}$, the static Nash equilibrium $(e, e)$ yields the minmax profit and thus constitutes the toughest punishment for both firms. When instead $e > \hat{p}$, each firm can only guarantee itself the incomplete-technology monopoly profit $\tilde{\pi}^m$, which is lower than the profit of the static Nash equilibrium $(\hat{p}, \hat{p})$; as shown by Lemma 3 below, Abreu's optimal penal codes may sustain the minmax profit, in which case it constitutes again the toughest punishment.

We will distinguish three cases, depending on the location of $e$ with respect to $p^m$ and $\hat{p}$.

## 3.1 Rivalry: $e < p^m$

Collusion can be profitable in the rivalry region only if competition is strong enough, as the loss in demand due to partial consumption grows with essentiality. For $e$ close to 0, this loss is small and the Nash profit is negligible; and so collusion, if feasible, is attractive for the firms. Conversely, for $e$ close to $p^m$, the Nash equilibrium approaches the highest possible profit $\pi^m$, whereas pricing above $e$ substantially reduces demand for the technology. When the patents are weak substitutes, in that $2\pi(e) > \tilde{\pi}^m(e)$, selling the incomplete technology reduces total profit. Because each firm can guarantee itself $\pi(e)$, there is no collusion (see online Appendix B for a more detailed proof):

**Lemma 2 (weak substitutes)** *Let $\underline{e} < p^m$ denote the unique solution to*

$$\tilde{\pi}^m(\underline{e}) = 2\pi(\underline{e}).$$

*When $e \in [\underline{e}, p^m]$, the unique equilibrium is the repetition of the static Nash one.*

When $e < \underline{e}$, raising the price above $p^N = e$ increases profits. Because users then buy only one license, each firm can attract all users by slightly undercutting the collusive price. Like in standard Bertrand oligopolies, *maximal* collusion (on $\tilde{p}^m(e)$) is sustainable whenever *some* collusion is sustainable. As symmetric collusion is easier to sustain, and deviations are optimally punished by reverting to static Nash behavior, such collusion is indeed sustainable if:

$$\frac{\tilde{\pi}^m(e)}{2} \geq (1-\delta)\,\tilde{\pi}^m(e) + \delta\pi(e),$$

leading to:

**Proposition 1 (rivalry)** *When $e < p^m$, tacit collusion is feasible if and only if*

$$\delta \geq \delta^N(e) \equiv \frac{1}{2}\frac{1}{1-\frac{\pi(e)}{\tilde{\pi}^m(e)}}. \quad (1)$$

*$\delta^N(e)$ is increasing in $e$ and exceeds 1 iff $e \geq \underline{e}$ (collusion is then never sustainable). If $e < \underline{e}$, the most profitable collusion occurs at price $\tilde{p}^m(e)$.*

**Proof.** See online Appendix C. ■

As the threshold $\delta^N(e)$ increases with $e$ in the range $e \in [0, p^m]$, for any given $\delta \in (1/2, 1)$, there exits a unique $\hat{e}(\delta) \in (0, \underline{e})$ such that collusion is feasible if and only if $e < \hat{e}(\delta)$: Greater essentiality hinders collusion, because the toughest punishment, given by the static Nash profit, becomes less effective as essentiality increases. Although the gains from deviation also decrease, which facilitates collusion, this effect is always dominated.

## 3.2 Weak complementors: $p^m < e \leq \hat{p}$

In the case of weak complementors, the static Nash equilibrium still yields minmax profits and thus remains the toughest punishment in case of deviation. Furthermore, like when $e \in [\underline{e}, p^m]$, selling the incomplete technology cannot be more profitable than the static Nash outcome.[8] Firms can however increase their profit by *lowering* their price below $p^N = e$. As $p_j \leq e < \hat{p} = r(\hat{p}) < r(p_j)$, firm $i$'s best deviation then consists in charging $e$. Hence, *perfect* cooperation on $p^m$ is sustainable when:

$$\pi^m \geq (1-\delta)\, eD(p^m + e) + \delta \pi(e), \tag{2}$$

which is satisfied for $\delta$ close enough to 1. Furthermore, when demand is convex, it can be checked that cooperation on some total price $P < 2e$ is easiest when it is symmetric (i.e., when $p_i = P/2$). Building on these insights leads to:

**Proposition 2 (weak complementors)** *When $p^m < e \leq \hat{p}$:*
*(i) Perfect cooperation on price $p^m$ is feasible if and only if*

$$\delta \geq \overline{\delta}^N(e) \equiv \frac{eD(p^m+e) - \pi^m}{eD(p^m+e) - \pi(e)},$$

*where $\overline{\delta}^N(e)$ lies strictly below 1 for $e > p^m$, and is decreasing for $e$ close to $p^m$.*
*(ii) Furthermore, if $D'' \geq 0$, then profitable cooperation on some stationary price is sustainable if and only if*

$$\delta \geq \underline{\delta}^N(e),$$

*where $\underline{\delta}^N(e)$ lies below $\overline{\delta}^N(e)$, is decreasing in $e$, and is equal to 0 for $e = \hat{p}$. The set of sustainable Nash-dominating per-firm payoffs is then $[\pi(e), \overline{\pi}(e,\delta)]$, where $\overline{\pi}(e,\delta) \in (\pi(e), \pi^m]$ is (weakly) increasing in $\delta$.*

**Proof.** See online Appendix D. ■

By contrast with the case of rivalry, where collusion inefficiently induces users to adopt the incomplete technology and may reduce profits, avoiding

[8] $\tilde{\pi}^m(e) = \tilde{p}^m(e)\, D(e + \tilde{p}^m(e)) < (e + \tilde{p}^m(e))\, D(e + \tilde{p}^m(e)) \leq 2eD(2e)$, where the first inequality stems from $e > 0$ and the second one from the fact that the aggregate profit $PD(P)$ is concave in $P$ and maximal for $P^m = 2p^m < 2e \leq e + \tilde{p}^m(e)$ (as $\tilde{p}^m(e) = r(e) \geq r(\hat{p}) = p \geq e$).

double marginalization unambiguously raise profits here. It follows that some cooperation (and even perfect cooperation) is always sustainable, for any degree of essentiality, when firms are sufficiently patient.

### 3.3 Strong complementors: $e > \hat{p}$

With strong complementors, the static Nash equilibrium $(\hat{p}, \hat{p})$ no longer yields the minmax payoff, equal here to the incomplete-technology monopoly profit: $\underline{\pi} = \tilde{\pi}^m(e)$. Abreu (1988)'s optimal penal codes then provide more severe punishments than the static Nash outcome. They have a particularly simple structure in the case of symmetric behaviors on- and off- the equilibrium path, as punishment paths then have two phases: a finite phase with a low payoff and then a return to the equilibrium cooperation phase.

**Lemma 3 (minmax with strong complementors)** *The minmax payoff is sustainable whenever*

$$\delta \geq \underline{\delta}(e) \equiv \frac{\tilde{\pi}^m(e) - \pi(e)}{\pi(\hat{p}) - \pi(e)},$$

*where* $\underline{\delta}(e) \in (0,1)$ *for* $e \in (\hat{p}, V)$, *and* $\underline{\delta}(V) = \lim_{e \longrightarrow \hat{p}} \underline{\delta}(e) = 0$.

**Proof.** See online Appendix E. ■

Optimal penal codes can sustain minmax profits not only when $e$ is close to $\hat{p}$ (where the static Nash yields minmax profits), but also in case of almost perfect complements (where $\tilde{\pi}^m(e)$ is close to 0): In this latter case, firms can sustain minmax profits by "choking-off" demand with prohibitive prices for a sufficiently long period of time before returning to cooperation.[9]

Following similar steps as for weak complementors, we can then establish:

**Proposition 3 (strong complementors)** *When* $e > \hat{p}$:
*i) Some profitable cooperation is always sustainable. Perfect cooperation on price* $p^m$ *is feasible if* $\delta \geq \overline{\delta}^N(e)$, *where* $\overline{\delta}^N(e)$ *continuously prolongs the function defined in Proposition 2, lies strictly below* 1, *and is decreasing for*

[9]The optimal deviation then consists in charging the monopoly price for the incomplete technology, but this coincides with the minmax.

*$e$ close to $\hat{p}$ and for $e$ close to $V$.*

*ii) Furthermore, if $D'' \geq 0$, then there exists $\overline{\pi}(e,\delta) \in (\pi(\hat{p}), \pi^m]$, which continuously prolongs the function defined in Proposition 2 and is (weakly) increasing in $\delta$, such that the set of Nash-dominating sustainable payoffs is $[\pi(\hat{p}), \overline{\pi}(e,\delta)]$.*

**Proof.** See online Appendix F. ■

With strong complementors, firms can sustain cooperation on some $p < p^N = \hat{p}$ for any discount factor: Starting from the static Nash price $\hat{p}$, a small reduction in the price generates a first-order increase in profits, and only a second-order incentive to deviate.

## 3.4 Summary and welfare analysis

Figure 2 summarizes the analysis so far.

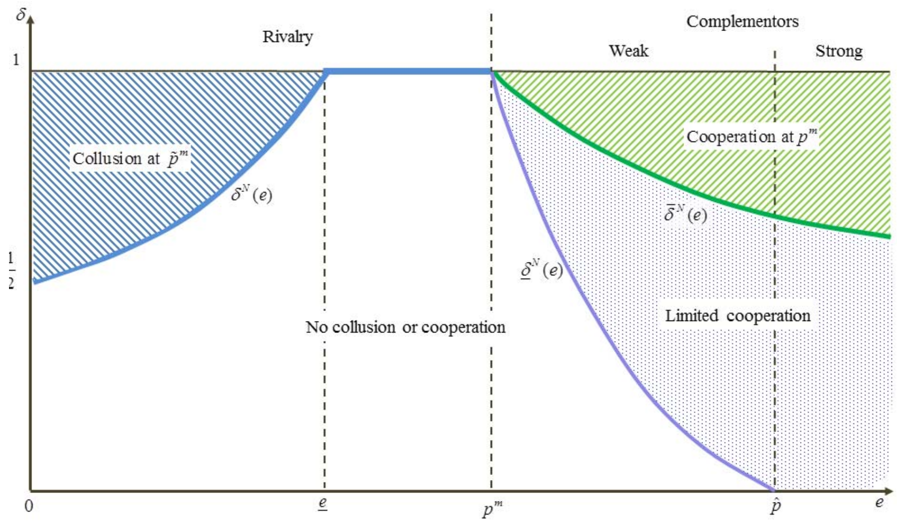


Figure 2: Tacit collusion and cooperation

Tacit coordination is easiest, and the gain from coordination highest, when the patents are close to being either perfect substitutes or perfect complements. Tacit coordination is impossible when patents are weak substitutes;

raising price then leads users to adopt an incomplete version of the technology, and decreases overall profit. Collusion by contrast is feasible when patents are strong substitutes, and all the more so as they become closer substitutes. Likewise, cooperation is not always feasible when patents are weak complementors, but the scope for cooperation increases as patents become more essential; finally, some cooperation is always possible when patents are strong complementors.

We now consider the impact of tacit coordination on users and society. To perform a welfare analysis we will assume that, whenever equilibria exist that are more profitable than the static Nash outcome, then firms coordinate on one of those equilibria.[10]

**Proposition 4 (welfare)** *Whenever firms coordinate on an equilibrium that is more profitable than the static Nash benchmark, such tacit coordination:*

*i) harms users and reduces total welfare under rivalry ($e < p^m$).*

*ii) benefits users and increases total welfare for complementors ($e > p^m$).*

**Proof.** When users acquire both licenses at total price $P$, welfare has the familiar expression:

$$W(P) = S(P) + PD(P),$$

where $S(P) \equiv \int_P^V D(\tilde{P})d\tilde{P}$. When instead users acquire a single license at price $p$, welfare is

$$\tilde{W}(p) = S(p+e) + pD(p+e).$$

Thus under rivalry ($e < p^m$), welfare is $W(2e)$ in the absence of collusion and $\tilde{W}(p)$ in the collusive outcome, for some $p > e$. Note that

$$\tilde{W}(p) = W(p+e) - eD(p+e).$$

This expression identifies the two facets of the collusive cost. First, the total price, $p + e$, exceeds the competitive price $2e$ as $p > e$. Second, there is

[10] We remain agnostic about equilibrium selection, as the conclusions hold for *any* profitable coordination.

a foregone surplus $e$ on actual consumption $D(p+e)$ due to incomplete consumption. Collusion harms consumers and reduces total welfare under rivalry.

In the case of complementors, tacit coordination is profitable when firms cooperate in offering the complete technology at a price lower than the static Nash price; it then benefits users and increases total welfare. ■

# 4 Joint marketing

We now assume that the firms are allowed to set up a pool, providing access to the whole technology at some price $P$. If the pool can forbid independent licensing, then the pool charges $P = P^m = 2p^m$ and each firm obtains $\pi^m$. From now on, we assume that, as is the case under current antitrust guidelines for technology transfers (in the US, Europe and Japan for example), independent licensing must be permitted by the pool. The pool can also offer licenses on a stand-alone basis if it chooses to. The pool further specifies a sharing rule for its dividends: Some fraction $\alpha_i \geq 0$ (with $\alpha_1 + \alpha_2 = 1$) goes to firm $i$.[11] The game thus operates as follows:

**1.** At date 0, the firms form a pool and fix three pool prices, $p_1^P$, and $p_2^P$ for the individual offerings, and $P^P \leq p_1^P + p_2^P$ for the bundle, as well as the dividend sharing rule.

**2.** Then at dates $t = 1, 2, ...$, the firms non-cooperatively set prices $p_i^t$ for their individual licenses; the profits of the pool are then shared according to the agreed rule.

## 4.1 Rivalry: $e < p^m$

The firms can of course collude as before, by not forming a pool or, equivalently, by setting pool prices at prohibitive levels ($P^P, p_i^P \geq V$, say); firms

[11] In our model, we can allow these shares to be contingent on the history of the sales made through the pool, either on a stand-alone basis or as part of the bundle, without altering the results. By contrast, when we mandate independent licensing we would not want to allow contracts that monitor sales of individual licenses and forces deviating firms to compensate the pool when they sell outside the pool.

then collude on selling the incomplete technology if $\delta \geq \delta^N(e)$. Alternatively, they can use the pool to sell the bundle at a higher price:

**Lemma 4** *In order to raise firms' profits, the pool must charge prices above the Nash price:* $\min\left\{\frac{P^P}{2}, p_1^P, p_2^P\right\} > e$.

**Proof.** See online Appendix G. ■

Thus, to be profitable, the pool must adopt a price $P^P > 2e$ for the complete technology, and a price $p_i^P > e$ for each license. This, in turn, implies that the repetition of static Nash outcome through independent licensing remains an equilibrium: If the other firm offers $p_j^t = e$ for all $t \geq 0$, buying an individual license from firm $j$ (corresponding to quality-adjusted total price $2e$) strictly dominates buying from the pool, and so the pool is irrelevant (firm $i$ will never receive any dividend from the pool); it is thus optimal for firm $i$ to set $p_i^t = e$ for all $t \geq 0$. Furthermore, this individual licensing equilibrium, which yields $\pi(e)$, still minmaxes all firms, as in every period each firm can secure $eD\left(e + \min\left\{e, p_j^t\right\}\right) \geq \pi(e)$ by undercutting the pool and offering an individual license at price $p_i^t = e$.

Suppose that tacit coordination enhances profits: $\bar{v} > \pi^N = \pi(e)$, where $\bar{v}$ denotes the maximal average discounted equilibrium payoff. In the associated equilibrium, there exists some period $\tau \geq 0$ in which the aggregate profit, $\pi_1^\tau + \pi_2^\tau$, is at least equal to $2\bar{v}$. If users buy an incomplete version of the technology in that period, then each firm can attract all users by undercutting the equilibrium price; the same reasoning as before then implies that collusion on $p_i^t = \tilde{p}^m$ is sustainable, and requires $\delta \geq \delta^N(e)$.

If instead users buy the complete technology in period $\tau$, then they must buy it from the pool,[12] and the per-patent price $p^P \equiv P^P/2$ must satisfy:

$$2\pi\left(p^P\right) = \pi_1^\tau + \pi_2^\tau \geq 2\bar{v} > 2\pi(e),$$

implying $p^P > e$. The best deviation then consists in offering an individual license at a price $p^D$ such that users are indifferent between buying the

[12] Users would combine individual licenses only if the latter were offered at prices not exceeding $e$; hence, the total price $P$ would not exceed $2e$. But $PD(P) = \pi_1^\tau + \pi_2^\tau \geq 2\bar{v} > 2\pi(e)$ implies $P > 2e$.

individual license and buying from the pool:

$$(V-e)-p^D=V-2p^P,$$

that is, $p^D=2p^P-e\,(>e)$; the highest deviation profit is therefore:

$$\pi^D=\left(2p^P-e\right)D\left(2p^P\right)=\pi\left(p\right)+\left(p^P-e\right)D\left(2p^P\right)>\pi\left(p^P\right).$$

Thus, the price $p^P$ is sustainable if there exists continuation payoffs $\left(v_1^{\tau+1},v_2^{\tau+1}\right)$ such that, for $i=1,2$:

$$(1-\delta)\,\pi_i^\tau+\delta v_i^{\tau+1}\geq(1-\delta)\left[\pi\left(p^P\right)+\left(p^P-e\right)D\left(2p^P\right)\right]+\delta\pi\left(e\right).$$

Adding these two conditions and using $\frac{v_1^{\tau+1}+v_2^{\tau+1}}{2}\leq\bar{v}\leq\frac{\pi_1^\tau+\pi_2^\tau}{2}=\pi\left(p^P\right)$:

$$\pi\left(p^P\right)\geq(1-\delta)[\pi\left(p^P\right)+\left(p^P-e\right)D\left(2p^P\right)]+\delta\pi\left(e\right). \tag{3}$$

Conversely, under this condition the pool price $p^P$ is *stable*: a bundle price $P^P=2p^P$, together with high enough individual prices ($p_i^P\geq p^P$, say) and an equal profit-sharing rule, ensures that no firm has an incentive to undercut the pool; each pool member thus obtains $\pi\left(p^P\right)$.

**Proposition 5 (pool in the rivalry region)** *Suppose $e\leq p^m$. As before, if $\delta\geq\delta^N(e)$ the firms can sell the incomplete technology at the monopoly price $\tilde{p}^m$ and share the associated profit, $\tilde{\pi}^m$. In addition, a per-license pool price $p^P$, yielding profit $\pi\left(p^P\right)$, is stable if* (3) *holds.*
*i) Perfect collusion (i.e., on a pool price $p^P=p^m$) is feasible if*

$$\delta\geq\bar{\delta}^P(e)\equiv\frac{1}{2-\frac{e}{p^m-e}\frac{D(2e)-D(2p^m)}{D(2p^m)}},$$

*where the threshold $\bar{\delta}^P(e)$ is increasing in $e$.*
*ii) If $D''\leq 0$, then some collusion (i.e., on a stable pool price $p^P\in(e,p^m]$)*

*is feasible if and only if*

$$\delta \geq \underline{\delta}^P(e) \equiv \frac{1}{2}\frac{1}{1+\frac{eD'(2e)}{D(2e)}}, \tag{4}$$

*where the threshold* $\underline{\delta}^P(e)$ *is also increasing in* $e$*.*

## 4.2 Weak or strong complementors: $p^m \leq e$

Forming a pool enables the holders of complementary patents to cooperate perfectly, by charging $P^m = 2p^m$ for the whole technology, *not* offering (along the collusive path) independent licenses, and sharing the profit equally. No deviation is then profitable: The best price for an individual license is $\tilde{p} = 2p^m - e$ (that is, the pool price minus a discount reflecting the essentiality of the foregone license), is here lower than $p^m$ (since $p^m \leq e$) and yields:

$$(2p^m - e)\,D(2p^m) < p^m D(2p^m) = \pi^m.$$

**Proposition 6 (pool with complements)** *With weak or strong complementors, a pool allows for perfect cooperation (even if independent licensing remains allowed) and gives each firm a profit equal to* $\pi^m$*.*

## 4.3 Do independent licenses screen in good pools and out bad ones?

The desirability of pools with independent licensing carries over under dynamic competition, with a caveat in the case of (weak) rivalry. In that case:

- Whenever collusion would be sustained in the absence of a pool, a pool can only benefit users. First, a pool has no effect if does not allow a more profitable collusion. Second, a pool benefits users if it enables a more profitable collusion, since users can then buy a license for the complete technology at a price $P \leq P^m = 2p^m$, which is preferable to buying a license for the incomplete technology at price $\tilde{p}^m(e)$: Because $r' > -1$,

$$e + \tilde{p}^m = e + r(e) > 0 + r(0) = P^m = 2p^m$$

• By contrast, when collusion could not be sustained in the absence of a pool, then a collusion-enabling pool harms users, who then face an increase in the price from $e$ to some $p > e$. Because $\underline{\delta}^P(0) = 1/2 = \delta^N(0)$ and $\overline{\delta}^P(p^m) = 1$, a sufficient condition for the collusion region to be larger under a pool than without a pool (i.e., $\underline{\delta}^P(e) < \delta^N(e)$) is that

$$\frac{\pi(e)}{\tilde{\pi}^m(e)} > -\frac{eD'(2e)}{D(2e)},$$

which holds for instance for linear demand.

• With weak or strong complementors, a pool enables perfect cooperation and benefits users as well as the firms: in the absence of the pool, the firms would either not cooperate and thus set $p = p^N = \min\{\hat{p}, e\} > p^m$, or cooperate and charge per-license price $p \in [p^m, p^N)$, as opposed to the (weakly) lower price, $p^m$, under a pool.

This leads to the following proposition:

**Proposition 7 (screening through independent licensing)** *Independent licensing provides a useful but imperfect screen:*
*i) A pool with independent licensing has no impact if* $\delta < \min\left\{\delta^N(e), \underline{\delta}^P(e)\right\}$ *in case of rivalry* $(e < p^m)$, *or if* $\delta \geq \overline{\delta}^N(e)$ *in case of complementors* $(e > p^m)$. *In all other cases, it increases profit and:*
- *in case of complementors, always lowers price;*
- *in case of rivalry, lowers the (quality-adjusted) price if* $\delta \geq \delta^N(e)$, *but raises price otherwise.*

*ii) Appending independent licensing to a pool is always welfare-enhancing; except for a low discount factor, however, it is not a perfect screen, as it allows firms producing weak substitutes to raise price when (and only when)* $\underline{\delta}^P(e) < \delta < \delta^N(e)$.

Figure 3 illustrates this analysis for a linear demand.

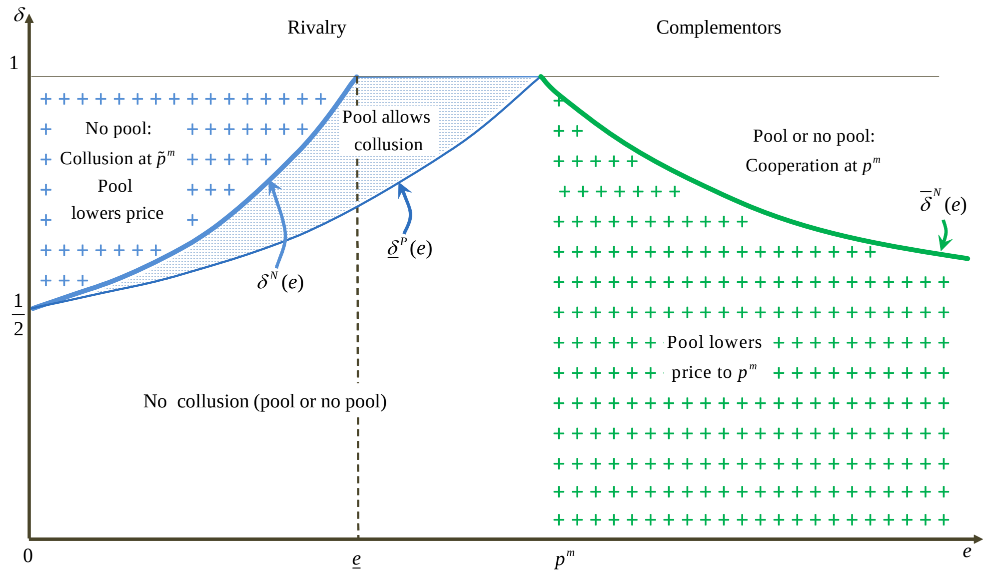


Figure 3: Impact of a pool
(+: beneficial; −: welfare reducing; blank: neutral)

# 5 A perfect screen for the validation of joint marketing alliances

The independent licensing requirement screens in good pools and out bad ones for a low enough discount factor. The requirement is then a perfect screen in that it enables a no-brainer approval of pools even in the absence of good estimates of essentiality.

With patient, coordinating firms, the independent licensing provision still does a reasonable job: it preserves the pool's ability to lower price under weak or strong complements; and with substitutes, it prevents the collusion inefficiency stemming from selling an incomplete technology at a high quality-adjusted price. However, as depicted in Figure 3, with weak substitutes there exists a region in which the pool can facilitate collusion. By eliminating the inefficiency from selling an incomplete technology (the corollary of an attempt to raise price in the absence of a pool), the pool makes high prices more attractive. Thus, authorities run the risk of approving a JMA of weak substitutes, generating some welfare loss along the way.

Tacit coordination thus poses a new challenge: Independent licensing no longer is a perfect screen. This section shows that another information-free instrument, namely, requiring unbundling and pass-through, can be appended so as to re-establish the perfect screen property, and that both instruments are needed to achieve this.

Under the unbundling and pass-through requirement:

1. The pool sets prices $p_1^P$ and $p_2^P$ at which the users can acquire individual licenses from the pool.[13] These act as price caps, as users can alternatively acquire individual licenses directly from their owner.

2. A firm's dividend is equal to the revenue generated by its technology; that is, firm $i$'s dividend is $p_i^P q_i^P$, where $q_i^P$ denotes the number of licenses of patent $i$ sold through the pool (as part of the bundle, or on a stand-alone basis).

In essence, and because we ignore transaction costs associated with multiple licenses, appending the unbundling & pass-through requirement to independent licensing makes the pool act merely as a price-cap setter. Each firm remains free to undercut the pool, but cannot sell its technology at a price exceeding the pool's price. The situation is thus formally the same as under independent licensing, but for the fact that each firm $i$ cannot charge more than $p_i^P$ for its technology.

From the proof of Lemma 2, to raise profit, the pool must charge individual prices above the static Nash level ($p_1^P, p_2^P > e$, and thus $P^P > P^N = 2e$), in order to sustain effective prices that are themselves above the static Nash level ($\underline{p}_1^t, \underline{p}_2^t > e$). But then, intuitively, a deviating member will aim at lowering its price rather than raising it, which in turn implies that profitable deviations are the same as in the absence of a pool. As the pool cannot make punishments more severe either (these are already maximal without a pool, as $\pi^N = \underline{\pi} = eD(2e)$), it follows that the pool cannot increase the scope for collusion.[14]

[13] Buying the bundle from the pool then costs $P^P = p_1^P + p_2^P$.

[14] There is a suspicion that price caps may facilitate collusion by providing focal points. The field evidence is however difficult to interpret, due to the lack of appropriate counterfactual – for instance, the fact that firms adhere to the ceilings can simply reflect that

By contrast, in the case of complementors, it is profitable to set the pool's prices below the static Nash level, so as to maintain effective prices closer to the cooperative level. Such prices also rule out any profitable deviation, as the deviator would need to raise the price above the pool's level.

**Proposition 8 (perfect screen)** *Appending the independent licensing and the unbundling & pass-through requirements to the pool*
*(i) has no impact on the outcome under a pool and therefore raises welfare relative to the absence of pool, if firms are complementors;*
*(ii) restores the no-pool outcome, thus making the pool welfare-neutral, under rivalry.*

**Proof.** See Appendix. ■

Note that the unbundling & pass-through requirement alone is an imperfect screen. For, consider the case of perfect substitutes; then in the absence of independent licenses, the firms can obtain the monopoly profit through a pool with unbundling (by setting $p_1^P = p_2^P = 2p^m$) even when the discount factor is low, whereas independent licensing guarantees perfectly competitive pricing.

# 6 Investment incentives

Allowing pools can also foster investment incentives. We now show that subjecting pools to independent licensing and unbundling ensures that pools benefit users, both through lower prices and by encouraging (only) value-adding investments, and increase total welfare – even in the presence of business stealing. To see this, suppose that initially only one piece of technology is available, and thus offered at a price $\tilde{p}^m(e)$, and consider an innovator's incentive to develop the other piece of technology.

these caps are effective in limiting double marginalization problems. Laboratory experiments have tried to circumvent this issue, and so far have failed to provide evidence of collusive, focal-point effects. See, e.g., Engelmann and Müller (2011) for a recent experiment designed to make collusion easier than in previous attempts, as well as a review of that literature.

**Proposition 9 (investment)** *Mandating independent licensing and unbundling & pass-through continues to make the pool welfare-neutral in case of rivalry, while allowing the pool to benefit users and society, both through lower prices and greater investment, in the case of complementors.*

**Proof.** See online Appendix I. ∎

When subject to independent licensing and unbundling, a pool does not affect investment in substitute products, as it has no impact on their prices; by contrast, by fostering cooperation among complementors the pool can enhance welfare not only through its impact on price and usage, but also by encouraging investment in such products – it can be checked that, despite some business stealing, the pool never generates more investment incentives than is socially desirable.

# 7 Extensions

## 7.1 Asymmetric offerings

Suppose now that essentiality differs across firms: The technology has value $V - e_i$ if the user buys only patent $j$ (for $i \neq j \in \{1, 2\}$); without loss of generality, suppose that $e_1 \geq e_2$.

*In the absence of tacit coordination*, firm $i$ solves (for $i \neq j \in \{1, 2\}$)

$$\max_{p_i \leq e_i} \{p_i D (p_j + p_i)\},$$

and thus charges $p_i = \min \{e_i, r(p_j)\}$. Therefore, the static Nash equilibrium $(p_1^N, p_2^N)$ can be of three types:[15]

- when both firms are constrained (i.e., $e_i \leq r(e_j)$), $(p_1^N, p_2^N) = (e_1, e_2)$;
- when only firm 2 is constrained ($e_1 > r(e_2)$ and $e_2 \leq r(r(e_2))$, which amounts to $e_2 < \hat{p}$), $(p_1^N, p_2^N) = (r(e_2), e_2)$;

[15] Note that $e_1 \geq e_2$ and $r' < 0$ imply $r(e_2) \geq r(e_1)$. Hence, $e_1 > r(e_2)$ implies $e_1 > r(e_2)$; as $r' > -1$, this in turn implies $e_1 + r(e_1) > r(e_1) + r(r(e_1))$, or $r(r(e_1)) < e_1$. Hence, the configuration $(p_1, p_2) = (e_1, r(e_1))$ cannot arise – that is, firm 1 cannot be constrained if firm 2 is not.

- when no firm is constrained ($e_1 > \hat{p}$ and $e_2 > \hat{p}$), $\left(p_1^N, p_2^N\right) = (\hat{p}, \hat{p})$.

*Rivalry* $\left(P^N < P^m\right)$ prevails when $e_1 + e_2 < P^m$,[16] in which case tacit coordination will aim at raising prices. Conversely, the two patents are *complementors* $\left(P^N > P^m\right)$ whenever either firm 1's best response is unconstrained or $e_1 + e_2 > P^m$; in this case tacit coordination will aim at reducing prices.

Combining the independent licensing and the unbundling & pass-through requirements still provides a perfect screen:

**Proposition 10 (asymmetric offerings)** *Appending the independent licensing and the unbundling & pass-through requirements to the pool provides a perfect screen even with asymmetric offerings: The pool*

*(i) allows firms to cooperate perfectly when they are complementors;*

*(ii) does not affect the scope for collusion under rivalry.*

**Proof.** See Appendix. ■

## 7.2 Multiple offerings

Suppose now that there are $n \geq 2$ symmetric firms: The technology has value $V(m)$ if the user buys $m \in \{1, ..., n\}$ patent(s), with $0 \leq V(1) \leq ... \leq V(n)$ and $V(n) > 0$. The demand for the bundle of $n$ patents at total price $P$ becomes

$$D(P) \equiv F(V(n) - P),$$

whereas the c.d.f. $F(.)$ satisfies the same regularity conditions as before (that is, Assumption A holds) and the essentiality parameter $e$ is now defined as the unique price satisfying

$$V(n) - ne = \max_{m \in \{1,...,n\}} \{V(m) - me\}.$$

Lerner and Tirole (2004) show that in the unique symmetric static Nash outcome users buy patents at price $p^N \equiv \min\{e, \hat{p}\}$, where $\hat{p}$ is now such

[16] This condition generalizes the one obtained under symmetry (namely, $e < p^m$, or $2e < P^m$) and implies $e_1 < r(e_2)$, $e_2 < r(e_1)$.

that

$$\hat{p} \equiv \arg\max_{p} \{pD(p + (n-1)\hat{p})\}.$$

Like under duopoly multiple marginalization implies $\hat{p} > p^m \equiv \arg\max_p \{npD(np)\}$, leading to three relevant regimes:

- Rivalry, when $e < p^m$; we then have $p^N = e$.
- Weak complementors, when $p^m < e < \hat{p}$; we then have again $p^N = e$.
- Strong complementors, when $e \geq \hat{p}$; we then have $p^N = \hat{p}$.

Our previous insights readily extend to any number of patents in the case of complementors.

**Proposition 11 (multiple complementors)** *When $e > p^m$, a pool achieves perfect cooperation, even if it is subject to independent licensing, by offering each patent at price $p_i^P = p^m$.*

**Proof.** It suffices to check that *not* offering independent licenses constitutes an equilibrium when pool members share the profit equally. A deviating firm must charge a price $p < p^m$ in order to sell licenses outside the pool, and by doing so it obtains

$$pD((n-1)p^m + p),$$

which increases with $p$ for $p \leq p^m$; hence there is no profitable deviation. ■

We now turn to the case of rivalry (i.e., $p^N = e < p^m$). Online Appendix K demonstrates the intuitive result that, raising total profit above the static Nash level requires selling an incomplete bundle:

**Lemma 5** *When $e < p^m$, generating more profit than the static Nash level requires selling less than $n$ products.*

For the sake of exposition, we now focus on symmetric equilibria sustained by Nash punishments, and show that pools cannot increase the scope for undesirable collusion:

**Proposition 12 (rivalry in $n$-firm oligopoly)** *When $e < p^m$, the set of symmetric, Nash dominating equilibrium outcomes that can be sustained by*

*Nash punishments is the same with or without a pool subject to independent licensing and unbundling.*

**Proof.** See online Appendix J. ■

Intuitively, firms must charge prices that induce users to buy only a subset of patents. But then, a firm cannot profitably deviate by raising its price, as it would exclude itself from the basket. Pool prices, acting as price caps, thus have no bite on profitable deviations, and so the pool cannot enhance the scope for collusion.

## 7.3 Strategic JMA

We have so far assumed that the JMA set prices once and for all. But consider the following possibility: Firms offer substitutes, and by simple majority (one vote if $n = 2$) can decide to set the bundle's price forever at $P = 0$; they do not exercise this give-away option as long as no-one deviates from the collusive price, but all vote for setting forever $P = 0$ after a deviation. Because each individual vote is irrelevant after a deviation (the technology will be given away regardless of one's vote), such behavior is individually optimal. And, crucially, payoffs below the no-JMA minmax can be enforced thanks to the pool. And so, more collusion can be sustained than in the absence of pool.

As implausible as this collective hara-kiri example sounds (if only because the firms employ a weakly dominated strategy when they vote in favor of giving away the technology), it makes the theoretical point that a JMA might be used to increase the discipline on members. One can react to this point in several ways. First, one can take it as a warning that antitrust authorities should keep an eye on instances in which a pool drastically lowers the price in reaction to low prices on individual licenses. Second, one can take a more ex-ante view of using regulation to prevent strategic pools from jeopardizing the information-free screens that were unveiled in this paper:

• *Charter regulation.* To prevent punishments below the minmax, it suffices to require unanimity among JMA members to change the pricing of joint offerings. In that case, each firm is able to guarantee itself $\pi(e)$ by refusing to renegotiate the initial deal. So, harsher punishments than the no-JMA minmax are infeasible.

- *Constraints on price flexibility.* In the stationary framework of this paper, prohibiting the JMA from adjusting its price prevents implementing such threats. This requirement however may be unreasonable in many instances, to the extent that members still face uncertainty about the nature of demand. Suppose for example that the initial demand for the technology results from distribution $F(\theta)$; with Poisson arrival rate $\lambda$, the final demand, corresponding to distribution $F(\theta+\omega)$, will be realized. Taking, say, the case of complementors, assuming that the hazard rate $f/F$ is decreasing, the JMA will want to raise (lower) its bundle price when $\omega$ is realized and positive (negative). Thus flexibility might be desirable. In this simple environment, one can imagine a rule allowing limited price flexibility through a ratchet: the price, once lowered, cannot be raised again. In this case, and ruling out weakly dominated strategies, no-one would vote for a pool price below the minmax outcome.

The broader lesson is of course that some antitrust attention must be paid to JMA enforcement of punishments. Such enforcements however seem rather easy to prevent, so we do not view them as a significant concern.

# 8 Concluding remarks

Competition policy guidelines and enforcement require a good understanding of factors that facilitate tacit collusion and cooperation. We were able to fully characterize optimal coordination among firms with an arbitrary pattern of rivalry or complementarity, parameterized in the context of our nested demand specification by one (or several) essentiality parameters. Coordination is easiest to achieve when offerings are strong substitutes or complements and most difficult in the intermediate range of weak rivalry and weak complementors.

The second and key contribution of the paper was the study of the treatment of joint marketing alliances. Antitrust authorities must be able to screen in good (price-reducing) JMAs and out bad (price-increasing) ones.

Such screening both reduces the deadweight loss of under-usage and provides innovators and potential entrants with incentives to bring to market essential new products, which create user value rather than steal rivals' business.

Authorities usually lack knowledge about the pattern of demand and therefore do not know whether such alliances are likely to be welfare-enhancing. It is therefore useful to devise acceptance rules that do not rely on authorities' fine knowledge about product essentiality. We considered two information-free requirements: independent licensing (maintained ownership and ability to market outside the JMA), and unbundling & pass-through (price caps).

We established two main results regarding JMAs subject to such requirements. While independent licensing is a perfect screen in the absence of tacit coordination, it is no longer so when rivals with weak substitutes take advantage of the JMA to raise price. But appending the unbundling & pass-through requirement re-establishes the perfect screen property.

Perfect screens not only ensure that JMAs correct the under-usage inefficiency in the right direction. They were shown to further affect investment incentives in the right way, in particular by boosting incentives to bring essential innovations to market.

While this analysis brings JMAs into safer territory, more research is desirable. The first area concerns generalizations of our nested demand function. While this specification affords much convenience, it also involves some restrictions. In particular, in the absence of separability between user characteristics and extent of essentiality, a JMA would want to engage in second-degree price discrimination so as to better extract user rents. The formulation and properties of the unbundling requirement would then be an interesting alley for research; as we know from Maskin-Riley (1984) and Mussa-Rosen (1978), non-linear pricing often involves price discounts and thus violates the unbundling requirement. And even in the absence of JMA, it would be worth producing a theory of tacit collusion in which consumers compose their basket à la carte, with different users selecting different baskets.

Second, the study of the so-called facilitating practices is a standard theme in antitrust economics. These include practices that enhance transparency, such as information exchanges through industry associations, as well as marketing practices such as advanced price announcements, product categorization that reduces the number of relevant prices, resale price maintenance, and so forth. The impact of such practices has been derived entirely in a perfect-substitutes world. Like in this paper, the extension to arbitrary degrees of substitutability or complementarity would much enhance our knowledge of their likely impact.

Finally, the identification of measures to prevent potentially detrimental effects of joint marketing also unveils an interesting alternative to mergers, whose analysis by antitrust authorities is well-known to be plagued by incomplete information. Mergers and JMAs differ in other dimensions than those studied in this paper, such as economizing on cost duplication or creating a different governance structure. We leave it to future research to determine whether and when JMAs should be considered by authorities as a superior approach to reaping the potential gains from mergers.

# Appendix: Proof of Propositions 8 and 10

Let us show that, with symmetric or asymmetric offerings, the independent licensing and the unbundling & pass-through requirements:

i) have no impact on the pool's strategy and operations, and therefore raises welfare relative to the absence of pool, if firms are complementors;

ii) restore the no-pool outcome, thus making the pool welfare-neutral, under rivalry.

The case of complementors (part i), where $e_1 + e_2 > P^m = 2p^m$) is straightforward: pool members can generate the cooperative profits, $2\pi^m$, by charging $P^P = P^m$ for the bundle. Under unbundling, adopting any prices $\left(p_1^P, p_2^P\right)$ satisfying $p_1^p + p_2^p = P^m$ and $p_i^p < e_i$ ensures that no user is attracted by the unbundled options; unbundling is irrelevant. Finally, no member has an incentive to undercut the pool's prices, as this would require offering $p_i < p_i^P$ and thus would yield $p_i D\left(p_j^P + p_i\right) < p_i^P D\left(p_j^P + p_i^P\right)$, as

$p_1^P + p_2^P = P^m = r(0) + 0 < p_j^P + r\left(p_j^P\right)$ implies $r\left(p_j^P\right) > p_i^P$, for $i \neq j \in \{1, 2\}$.[17]

We now turn to the case of rivalry (part ii), where $e_1 + e_2 < P^m$). We have already noted in the text that the pool must charge prices above the static Nash level to be profitable. Furthermore, offering a price $p_i^P > V$ would be irrelevant. Thus, without loss of generality, suppose now that the pool sets a price $p_i^P \in (e_i, V]$ for each patent $i = 1, 2$ (together with a price $P^P \geq p_1^P + p_2^P$ for the bundle). We first show that the minmax profits: (a) are the same as without a pool, and (b) can be sustained by the repetition of the static Nash outcome, $(e_1, e_2)$. To establish (a), it suffices to note that the minmaxing strategy $p_j = e_j$ remains available to firm $i$'s rival, and firm $i$'s best response, $p_i = e_i$, also remains available: by setting a price $p_k = e_k$, firm $k \in \{1, 2\}$ generates an effective price $\underline{p}_k \equiv \min\left\{p_k, p_k^P\right\} = e_k$. To establish (b), it suffices to note that the static Nash outcome $(e_1, e_2)$ remains feasible, and that deviations are only more limited than in the absence of a pool (as the firms can no longer induce an effective price higher than the pool's price for their patents).

We now show that any collusion sustainable with the pool is also sustainable without a pool. For the sake of exposition, we restrict attention to pure-strategy equilibria, but the reasoning extends to mixed strategies,[18] at the cost of significant notational complexity. Recall that the set of pure-strategy equilibrium payoffs can be characterized as the largest self-generating set of payoffs, where, as minmax profits are sustainable, a self-generating set of payoffs $W$ is such that, for any payoff $(\pi_1, \pi_2)$ in $W$, there exists a continuation payoff $(\pi_1^*, \pi_2^*)$ in $W$ and a price profile $(p_1^*, p_2^*) \in \mathcal{P}_1 \times \mathcal{P}_2$, where $\mathcal{P}_i$ is the set of admissible prices for firm $i$ (more on this below), that satisfy, for

[17] Without the pass-through requirement, a firm might have an incentive to undercut the pool, so as to secure all the revenue from its technology, and also obtain a share in the pool's revenue from the other technology.

[18] It is straightforward to apply the reasoning to public mixed strategies (i.e., when players randomize on the basis on public signals). In case of private mixed strategies (where only the realization is observed by the other players, and there is thus imperfect monitoring), the reasoning applies to public perfect equilibria, where strategies are based on public history (i.e., players do not condition their future decisions on their private choice of a lottery, but only on its realization). See e.g., Mailath and Samuelson (2006), Chapter 2.

$i \neq j \in \{1, 2\}$:

$$\pi_i = (1 - \delta)\, \pi_i \left(p_i^*, p_j^*\right) + \delta \pi_i^* \geq \max_{p_i \in \mathcal{P}_i} \pi_i \left(p_i, p_j^*\right) + \delta \underline{\pi}_i, \tag{5}$$

where $\pi_i (p_i, p_j)$ denotes firm $i$'s profit when the two patents are offered at prices $p_1$ and $p_2$. To establish that the equilibrium payoffs generated by a pool are also equilibrium payoffs without the pool, it suffices to show that any self-generating set in the former situation is also a self-generating set in the latter situation.

In the absence of a pool, without loss of generality the set of admissible prices for firm $i$ is $\mathcal{P}_i^N \equiv [0, V]$; when the pool offers a price $p_i^P$ for patent $i$, then the admissible set for the *effective price* $\underline{p}_i = \min \left\{p_i, p_i^P\right\}$ is $\mathcal{P}_i^P \equiv \left[0, p_i^P\right]$. Consider now a self-generating set $W^P$ for given pool prices $\left(p_1^P, p_2^P\right)$ satisfying $p_i^P \in (e_i, V]$ for $i = 1, 2$, and a given payoff $(\pi_1, \pi_2) \in W^P$, with associated payoff $(\pi_1^*, \pi_2^*) \in W^P$ and effective price profile $\left(\underline{p}_1^*, \underline{p}_2^*\right) \in \mathcal{P}_1^P \times \mathcal{P}_2^P$ satisfying

$$\pi_i = (1 - \delta)\, \pi_i \left(\underline{p}_i^*, \underline{p}_j^*\right) + \delta \pi_i^* \geq \max_{\underline{p}_i \in \mathcal{P}_i^P} \pi_i \left(\underline{p}_i, \underline{p}_j^*\right) + \delta \underline{\pi}_i. \tag{6}$$

By construction, the associated price profile $\left(\underline{p}_1^*, \underline{p}_2^*\right)$ also belongs to $\mathcal{P}_1^N \times \mathcal{P}_2^N$. However, the gain from a deviation may be lower than in the absence of a pool, as the set of admissible deviating prices is smaller. To conclude the proof, we now show that, for any $\left(\underline{p}_1^*, \underline{p}_2^*\right) \in \mathcal{P}_1^P \times \mathcal{P}_2^P$ satisfying (5), there exists $(p_1^*, p_2^*) \in \mathcal{P}_1^N \times \mathcal{P}_2^N$ satisfying

$$\pi_i = (1 - \delta)\, \pi_i \left(p_i^*, p_j^*\right) + \delta \pi_i^* \geq \max_{p_i \in \mathcal{P}_i^N} \pi_i \left(p_i, p_j^*\right) + \delta \underline{\pi}_i. \tag{7}$$

For this, it suffices to exhibit a profile $(p_1^*, p_2^*) \in \mathcal{P}_1^N \times \mathcal{P}_2^N$ yielding the same profits (i.e., $\pi_i \left(p_i^*, p_j^*\right) = \pi_i \left(\underline{p}_i^*, \underline{p}_j^*\right)$ for $i = 1, 2$) without increasing the scope for deviations (i.e., $\max_{p_i \in \mathcal{P}_i^N} \pi_i \left(p_i, p_j^*\right) \leq \max_{\underline{p}_i \in \mathcal{P}_i^P} \pi_i \left(\underline{p}_i, \underline{p}_j^*\right)$ for $i = 1, 2$). We can distinguish four cases for the associated price profile $\left(\underline{p}_1^*, \underline{p}_2^*\right)$:

*Case a:* $\underline{p}_1^* \leq e_1, \underline{p}_2^* \leq e_2$. In that case, we can pick $(p_1^*, p_2^*) = \left(\underline{p}_1^*, \underline{p}_2^*\right)$; as firm $i$'s profit from deviating to $p_i$ is then given by

$$\pi_i\left(p_i,\underline{p}_j^*\right)=\begin{cases} p_iD\left(\underline{p}_j^*+p_i\right) \text{ if } p_i\le e_i \\ 0 \text{ otherwise}\end{cases},$$

the best deviation is

$$\arg\max_{p_i\le e_i} p_iD\left(\underline{p}_j^*+p_i\right)=e_i,$$

which belongs to both $\mathcal{P}_i^N$ and $\mathcal{P}_i^P$. Hence, $\max_{\underline{p}_i\in\mathcal{P}_i^P}\pi_i\left(\underline{p}_i,\underline{p}_j^*\right)=\max_{p_i\in\mathcal{P}_i^N}\pi_i\left(p_i,\underline{p}_j^*\right)$.

*Case b:* $\underline{p}_i^*-e_i\le 0<\underline{p}_j^*-e_j$, for $i\ne j\in\{1,2\}$. In that case, the profile $\left(\underline{p}_1^*,\underline{p}_2^*\right)$ yields profits $\pi_j\left(\underline{p}_j^*,\underline{p}_i^*\right)=0$ and $\pi_i\left(\underline{p}_i^*,\underline{p}_j^*\right)=\underline{p}_i^*D\left(e_j+\underline{p}_i^*\right)$, and best deviations are respectively given by:

$$\begin{aligned}\arg\max_{p_j}\pi_j\left(p_j,\underline{p}_i^*\right) &= \arg\max_{p_j\le e_j} p_jD\left(\underline{p}_i^*+p_j\right)=e_j,\\ \arg\max_{p_i}\pi_i\left(p_i,\underline{p}_j^*\right) &= \arg\max_{p_i\le\underline{p}_j^*+e_i-e_j} p_iD\left(e_j+p_i\right)=\min\left\{\underline{p}_j^*+e_i-e_j,p_i^m\right\}.\end{aligned}$$

As $e_j\in\mathcal{P}_j^N\cap\mathcal{P}_j^P$, $\max_{\underline{p}_j\in\mathcal{P}_j^P}\pi_j\left(\underline{p}_j,\underline{p}_i^*\right)=\max_{p_j\in\mathcal{P}_j^N}\pi_j\left(p_j,\underline{p}_i^*\right)$. Therefore, if $\min\left\{\underline{p}_j^*+e_i-e_j,p_i^m\right\}\le p_i^P$ (and thus $\min\left\{\underline{p}_j^*+e_i-e_j,p_i^m\right\}\in\mathcal{P}_i^N\cap\mathcal{P}_i^P$), we can pick $(p_1^*,p_2^*)=\left(\underline{p}_1^*,\underline{p}_2^*\right)$, as then we also have $\max_{\underline{p}_i\in\mathcal{P}_i^P}\pi_i\left(\underline{p}_i,\underline{p}_j^*\right)=\max_{p_i\in\mathcal{P}_i^N}\pi_i\left(p_i,\underline{p}_j^*\right)$. If instead $\min\left\{\underline{p}_j^*+e_i-e_j,p_i^m\right\}>p_i^P$, then we can pick $p_i^*=\underline{p}_i^*$ and $p_j^*\in\left(e_j,e_j+p_i^P-e_i\right)$:[19] the profile $(p_1^*,p_2^*)$ yields the same profits as $\left(\underline{p}_1^*,\underline{p}_2^*\right)$, and, as the best deviations are the same, with or without the pool:

$$\begin{aligned}\arg\max_{p_j}\pi_j\left(p_j,p_i^*\right) &= \arg\max_{p_j}\pi_j\left(p_j,\underline{p}_i^*\right)=e_j\in\mathcal{P}_j^N\cap\mathcal{P}_j^P,\\ \arg\max_{p_i}\pi_i\left(p_i,p_j^*\right) &= \arg\max_{p_i\le p_j^*+e_i-e_j} p_iD\left(e_j+p_i\right)=\min\left\{p_j^*+e_i-e_j,p_i^m\right\}\in\mathcal{P}_i^N\cap\mathcal{P}_i^P,\end{aligned}$$

as $\min\left\{p_j^*+e_i-e_j,p_i^m\right\}\le p_j^*+e_i-e_j<p_i^P$.

[19] This interval is not empty, as $p_i^P>e_i$ by assumption.

*Case c:* $0 < \underline{p}_i^* - e_i = \underline{p}_2^* - e_2$. In that case, we can pick $(p_1^*, p_2^*) = \left(\underline{p}_1^*, \underline{p}_2^*\right)$, as best deviations consist in undercutting the other firm, and this is feasible with or without the pool.

*Case d:* $0 < \underline{p}_1^* - e_1 < \underline{p}_j^* - e_j$, for $i \neq j \in \{1, 2\}$. In that case, the same payoff could be sustained with $p_i^* = \underline{p}_i^*$ and $p_j^* = \underline{p}_i^* + e_j - e_i \left(< \underline{p}_j^*\right)$, with the convention that technology adopters, being indifferent between buying a single license from $i$ or from $j$, all favor $i$: the profile $(p_1^*, p_2^*)$ yields the same profits as $\left(\underline{p}_1^*, \underline{p}_2^*\right)$, $\pi_j = 0$ and $\pi_i = \underline{p}_i^* D\left(e_j + \underline{p}_i^*\right)$, but reduces the scope for deviations, which now boil down to undercutting the rival:

$$\begin{aligned}
\max_{p_j \in \mathcal{P}_j^N} \pi_j\left(p_j, p_i^*\right) &= \max_{\underline{p}_j \in \mathcal{P}_j^P} \pi_j\left(\underline{p}_j, \underline{p}_i^*\right) = \max_{p_j \leq \underline{p}_i^* + e_j - e_i} p_j D\left(e_j + p_j\right), \\
\max_{p_i \in \mathcal{P}_i^N} \pi_i\left(p_i, p_j^*\right) &= \max_{p_i \leq p_j^* + e_i - e_j} p_i D\left(e_j + p_i\right) \leq \max_{\underline{p}_i \in \mathcal{P}_i^P} \pi_i\left(\underline{p}_i, \underline{p}_j^*\right) = \max_{p_i \leq \underline{p}_j^* + e_i - e_j} p_i D\left(e_j + p_i\right).
\end{aligned}$$

This moreover implies that, as in case c above, these best deviations were already feasible with the pool. Indeed, as $p_k^* = p_h^* + e_k - e_h$, for $h \neq k \in \{1, 2\}$, we have:

$$\begin{aligned}
\arg\max_{p_j} \pi_j\left(p_j, p_i^*\right) &= \arg\max_{\underline{p}_j} \pi_j\left(\underline{p}_j, \underline{p}_i^*\right) = \arg \max_{p_j \leq \underline{p}_i^* + e_j - e_i} p_j D\left(e_j + p_j\right) = \min\left\{p_j^*, p_j^m\right\}, \\
\arg\max_{p_i} \pi_i\left(p_i, p_j^*\right) &= \arg \max_{p_i \leq p_j^* + e_i - e_j} p_i D\left(e_j + p_i\right) = \min\left\{p_i^*, p_i^m\right\},
\end{aligned}$$

where $\min\left\{p_j^*, p_j^m\right\} \in \mathcal{P}_j^N \cap \mathcal{P}_j^P$, as $\min\left\{p_j^*, p_j^m\right\} \leq p_j^* < \underline{p}_j^* \in \mathcal{P}_j^P \left(\subset \mathcal{P}_j^N\right)$, and likewise $\min\left\{p_i^*, p_i^m\right\} \in \mathcal{P}_i^N \cap \mathcal{P}_i^P$, as $\min\left\{p_i^*, p_i^m\right\} \leq p_i^* = \underline{p}_i^* \in \mathcal{P}_i^P \left(\subset \mathcal{P}_i^N\right)$.

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# Online Appendix
# Not for publication

## A Analysis of best response functions

The function $r(p)$ is characterized by the FOC

$$D(p + r(p)) + r(p) D'(p + r(p)) = 0,$$

and thus:

$$r'(p) = -\frac{D'(p + r(p)) + r(p) D''(p + r(p))}{2D'(p + r(p)) + r(p) D''(p + r(p))},$$

where the numerator and the denominator are both negative under Assumption A: this is obvious is $D''(p + r(p)) \leq 0$ and, if $D''(p + r(p)) > 0$, then

$$D'(p + r(p)) + r(p) D''(p + r(p)) < D'(p + r(p)) + (p + r(p)) D''(p + r(p)),$$

where the right-hand side is negative under Assumption A; therefore, $r'(p) < 0$. Since the denominator is obviously more negative, we also have $r'(p) > -1$.

We now use this function to characterize $p^r(p)$, the actual best response of an firm to its rival's price $p$. If the rival charges a price $p \leq e$, then users (weakly) favor relying on both technologies whenever they buy the firm's own technology, and they are willing to do so as long only if the firm does not charge more than $e$; therefore, the firm's best response is given by:

$$p^r(p) = \arg\max_{\tilde{p} \leq e} \tilde{p} D(p + \tilde{p}) = \min\{r(p), e\}.$$

If instead the rival charges a price $p > e$, then users use at most one technology, and look for the lowest price; therefore, the firm's best response is given by:

$$p^r(p) = \arg\max_{\tilde{p} < p} \tilde{\pi}(\tilde{p}) = \arg\max_{\tilde{p} < p} \tilde{p} D(e + \tilde{p}) = \min\{r(e), p_-\},$$

where $p_-$ stands for slightly undercutting the rival's price $p$. It follows that the Nash equilibrium is unique, symmetric, and consists for both firms in charging $p^N = \{e, \hat{p}\}$; indeed:

- if $e \geq \hat{p}$, then $\hat{p} = r(\hat{p}) \leq e$ implies $p^r(\hat{p}) = \hat{p}$;
- if instead $e < \hat{p}$, then $r(e) > \hat{p} > e$ implies $p^r(e) = e$.

Finally, define $P^m \equiv 2p^m$ and $\hat{P} \equiv 2\hat{p}$. By construction, we have

$$P^m = 2p^m = \arg\max_P PD(P),$$

whereas

$$\hat{p} = r(p) = \arg\max_p pD(\hat{p} + p),$$

implies

$$\hat{P} = \arg\max_P (P - \hat{p}) D(P).$$

A revealed preference argument then implies $\hat{P} > P^m$, and thus $\hat{p} > p^m$.

# B Proof of Lemma 2

Let $\pi_i(p_i, p_j)$ denote firm $i$'s profit. Prices such that $\min\{p_1, p_2\} \leq e$ cannot yield greater profits than the static Nash:

- If $p_1, p_2 \leq e$, total price $P$ is below $2e$; as the aggregate profit $PD(P)$ is concave in $P$ and maximal for $P^m = 2p^m > 2e$, total profit is smaller than the Nash level.
- If instead $p_i \leq e < p_j$, then

  $$\pi_1(p_1, p_2) + \pi_2(p_2, p_1) = p_i D(e + p_i) \leq eD(2e) \leq 2eD(2e) = 2\pi^N,$$

  where the first inequality stems from the fact that the profit $\tilde{\pi}(p) = pD(e + p)$ is concave in $p$ and maximal for $\tilde{p}^m(e) = r(e)$, which exceeds $e$ in the rivalry case (as then $e < p^m < \hat{p} = r(\hat{p})$).

Therefore, to generate more profits than the static Nash profit in a given period, both firms must charge more than $e$; this, in turn, implies that users buy at most one license, and thus aggregate profits cannot exceed $\tilde{\pi}^m(e)$. It follows that collusion cannot enhance profits if $\tilde{\pi}^m(e) \leq 2\pi^N = 2\pi(e)$. Keeping $V$ and thus $p^m$ constant, increasing $e$ from 0 to $p^m$ decreases $\tilde{\pi}^m(e) = \max_p pD(p+e)$ but increases $\pi(e)$; as $\tilde{\pi}^m(0) = 2\pi(p^m) = 2\pi^m$, there exists a unique $\underline{e} < p^m$ such that, in the range $e \in [0, p^m]$, $\tilde{\pi}^m(e) < 2\pi^N$ if and only if $e > \underline{e}$.

Thus, when $e > \underline{e}$, the static Nash payoff $\pi^N$ constitutes an upper bound on average discounted equilibrium payoffs. But the static Nash equilibrium here yields minmax profits, and thus also constitutes a lower bound on equilibrium payoffs. Hence, $\pi^N$ is the unique average discounted equilibrium payoff, which in turn implies that the static Nash outcome must be played along any equilibrium path.

# C Proof of Proposition 1

Suppose that collusion raises profits: $\bar{v} > \pi^N$, where, recall, $\bar{v}$ is the maximal average discounted equilibrium payoff. As $\bar{v}$ is a weighted average of per-period profits, along the associated equilibrium path there must exist some period $\tau \geq 0$ in which the aggregate profit, $\pi_1^\tau + \pi_2^\tau$, is at least equal to $2\bar{v}$. This, in turn, implies that users must buy an incomplete version of the technology; thus, there exists $\bar{p}$ such that:

$$\tilde{\pi}(\bar{p}) = \pi_1^\tau + \pi_2^\tau \geq 2\bar{v}.$$

By undercutting its rival, each firm $i$ can obtain the whole profit $\tilde{\pi}(\bar{p})$ in that period; as this deviation could at most be punished by reverting forever to the static Nash behavior, a necessary equilibrium condition is, for $i = 1, 2$:

$$(1-\delta)\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta)\tilde{\pi}(\bar{p}) + \delta\underline{\pi},$$

where $v_i^{\tau+1}$ denotes firm $i$'s continuation equilibrium payoff from period $\tau+1$ onwards. Adding these conditions for the two firms yields:

$$(1-\delta)\,\tilde{\pi}(\bar{p})+\delta\underline{\pi}\leq(1-\delta)\,\frac{\pi_1^{\tau}+\pi_2^{\tau}}{2}+\delta\frac{v_1^{\tau+1}+v_2^{\tau+1}}{2}\leq(1-\delta)\,\frac{\tilde{\pi}(\bar{p})}{2}+\delta\frac{\tilde{\pi}(\bar{p})}{2},$$

where the second inequality stems from $v_1^{\tau+1}+v_2^{\tau+1}\leq 2\overline{v}\leq\pi_1^{\tau}+\pi_2^{\tau}=\tilde{\pi}(\bar{p})$. This condition amounts to

$$\left(\delta-\frac{1}{2}\right)\tilde{\pi}(\bar{p})\geq\delta\underline{\pi}=\delta\pi(e)\,, \tag{8}$$

which requires $\delta\geq 1/2$ (with a strict inequality if $e>0$). This, in turn, implies that (8) must hold for $\tilde{\pi}^m(e)=\max_{\bar{p}}\tilde{\pi}(\bar{p})$:

$$\left(\delta-\frac{1}{2}\right)\tilde{\pi}^m(e)\geq\delta\pi(e)\,. \tag{9}$$

Conversely, if (9) is satisfied, then the stationary path $(\tilde{p}^m(e),\tilde{p}^m(e))$ (with equal market shares) is an equilibrium path, as the threat of reverting to the static Nash behavior ensures that no firm has an incentive to deviate:

$$\frac{\tilde{\pi}^m(e)}{2}\geq(1-\delta)\,\tilde{\pi}^m(e)+\delta\pi(e)\,,$$

or

$$\delta\geq\delta^N(e)\equiv\frac{1}{2}\frac{1}{1-\frac{\pi(e)}{\tilde{\pi}^m(e)}}.$$

Finally, $\delta^N(e)$ increases with $e$, as $\pi(e)$ increases with $e$ in that range, whereas $\tilde{\pi}^m(e)=\max_p\{pD(p+e)\}$ decreases as $e$ increases.

# D Proof of Proposition 2

*i)* That perfect cooperation (on $p_i^t=p^m$ for $i=1,2$ and $t=0,1,...$) is sustainable if and only if

$$\delta\geq\overline{\delta}^N(e)=\frac{eD(p^m+e)-\pi^m}{eD(p^m+e)-\pi(e)}=\frac{1}{1+\frac{\pi^m-\pi(e)}{eD(p^m+e)-\pi^m}}$$

derives directly from (2).

For $e \in (p^m, \hat{p}]$, $\pi^m > \pi(e)$ and $eD(p^m + e) > \pi^m$ (as $r(p^m) > r(e) \geq \hat{p} \geq e$); therefore, $\overline{\delta}^N(e) < 1$. Also, for $\varepsilon$ positive but small, we have:

$$\overline{\delta}^N(p^m + \varepsilon) \simeq \frac{1}{1 - \frac{\pi''(p^m)}{D(2p^m)+p^m D'(2p^m)} \frac{\varepsilon}{2}},$$

which decreases with $\varepsilon$, as $\pi''(p^m) < 0$ and

$$D(2p^m) + p^m D'(2p^m) = -p^m D'(2p^m) > 0.$$

*ii)* Suppose that collusion enhances profits: $\bar{v} > \pi^N = \pi(e)$. In the most profitable collusive equilibrium, there exists again some period $\tau$ in which the average profit is at least $\bar{v}$. And as $\bar{v} > \pi(e) > \tilde{\pi}^m(e)$, users must buy the complete technology in that period; thus, each firm $i$ must charge a price $p_i^\tau$ not exceeding $e$, and the average price $\bar{p} = \frac{p_1^\tau + p_2^\tau}{2}$ must moreover satisfy

$$\pi(\bar{p}) = \frac{\pi_1^\tau + \pi_2^\tau}{2} \geq \bar{v}.$$

As $p_j \leq e < \hat{p} = r(\hat{p}) < r(p_j)$, firm $i$'s best deviation consists in charging $e$. Hence, to ensure that firm $i$ has no incentive to deviate, we must have:

$$(1-\delta)\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta) eD\left(p_j^\tau + e\right) + \delta\underline{\pi}.$$

Combining these conditions for the two firms yields, using $\pi(\bar{p}) = \frac{\pi_1^\tau + \pi_2^\tau}{2}$ and $\underline{\pi} = \pi(e)$:

$$(1-\delta) e\frac{D(p_1^\tau + e) + D(p_2^\tau + e)}{2} + \delta\pi(e) \leq (1-\delta)\pi(\bar{p}) + \delta\frac{v_2^{\tau+1} + v_2^{\tau+1}}{2} \leq \pi(\bar{p}),$$

where the second inequality stems from $\frac{v_1^{\tau+1} + v_2^{\tau+1}}{2} \leq \bar{v} \leq \pi(\bar{p})$. If the demand function is (weakly) convex (i.e., $D'' \geq 0$ whenever $D > 0$), then this condition implies $H(\bar{p}; e, \delta) \geq 0$, where

$$H(p; e, \delta) \equiv \pi(p) - (1-\delta) eD(p+e) - \delta\pi(e). \tag{10}$$

Conversely, if $H(\bar{p}; e, \delta) \geq 0$, then the stationary path $(\bar{p}, \bar{p})$ is an equilibrium path.

Summing-up, when $D'' \geq 0$, some profitable cooperation is feasible (i.e., $\bar{v} > \pi^N$) if and only if there exists $\bar{p} < e$ satisfying $\pi(\bar{p}) > 2\pi^N$ and $H(\bar{p}; e, \delta) \geq 0$. By construction, $H(e; e, \delta) = 0$. In addition,

$$\frac{\partial H}{\partial p}(p; e, \delta) = D(2p) + 2pD'(2p) - (1-\delta)\, eD'(p+e).$$

Hence, $D'' \geq 0$ and Assumption A (which implies that $PD'(P)$ decreases with $P$) ensure that

$$\frac{\partial^2 H}{\partial p^2}(p; e, \delta) < 0.$$

Therefore, if $J(e, \delta) \geq 0$, where:

$$J(e,\delta) \equiv \frac{\partial H}{\partial p}(e; e, \delta) = D(2e) + (1+\delta)\, eD'(2e),$$

then no cooperation is feasible, as then $H(p; e, \delta) < 0$ for $p < e$. Conversely, if $J(e, \delta) < 0$, then tacit cooperation on $\bar{p}$ is feasible for $\bar{p} \in \left[\underline{p}(e, \delta), e\right]$, where $p = \underline{p}(e, \delta)$ is the unique solution (other than $p = e$) to $H(p; e, \delta) = 0$. Note that

$$\frac{\partial J}{\partial \delta}(e, \delta) = eD'(2e) < 0,$$

and

$$J(e, 0) = D(2e) + eD'(2e) \geq 0,$$

as $e \leq \hat{p} \leq r(e)$, whereas

$$J(e, 1) = D(2e) + 2eD'(2e) < 0,$$

as $e > p^m$. Therefore, there exists a unique $\underline{\delta}^N(e)$ such that tacit cooperation can be profitable for $\delta > \underline{\delta}^N(e)$. Furthermore, Assumption A implies that $eD'(2e)$ is decreasing and so

$$\frac{\partial J}{\partial e}(e, \delta) = 2D'(2e) + (1+\delta)\frac{d}{de}(eD'(2e)) < 0.$$

Hence the threshold $\underline{\delta}^N(e)$ decreases with $e$; furthermore, $\underline{\delta}^N(\hat{p}) = 0$, as $J(\hat{p}, 0) = D(2\hat{p}) + \hat{p}D'(2\hat{p}) = 0$ (as $\hat{p} = r(\hat{p})$).

Finally, when $\delta > \underline{\delta}^N(e)$, the set of sustainable Nash-dominating per-firm payoffs is $[\pi(e), \overline{\pi}(e,\delta)]$, where $\overline{\pi}(e,\delta) \equiv \pi\left(\max\left\{p^m, \underline{p}(e,\delta)\right\}\right)$, and $\underline{p}(e,\delta)$ is the lower solution to $H(p; e, \delta) = 0$; as $H$ increases in $\delta$,[20] $\underline{p}(e,\delta)$ decreases with $\delta$ and thus $\overline{\pi}(e,\delta)$ weakly increases with $\delta$.

# E Proof of Lemma 3

In order to sustain the minmax profit $\underline{\pi} = \tilde{\pi}^m(e)$, consider the following two-phase, symmetric penal code. In the first phase (periods $t = 1, ..., T$ for some $T \geq 1$), both firms charge $e$, so that the profit is equal to $\pi(e)$. In the first period of the second phase (i.e., period $T+1$), with probability $1-x$ both firms charge $e$, and with probability $x$ they switch to the best collusive price that can be sustained with minmax punishments, which is defined as:

$$p^C(e,\delta) \equiv \arg\max_p pD(2p),$$

subject to the constraint

$$(1-\delta)\max_{\tilde{p}\leq e} \tilde{p}D(p+\tilde{p}) + \delta\underline{\pi} \leq pD(2p). \quad (11)$$

Then, in all following periods, both firms charge $p^C$. Letting $\Delta = (1-\delta)x\delta^T + \delta^{T+1} \in (0,\delta)$ denote the fraction of (discounted) time in the second phase, the average discounted per-period punishment profit is equal to

$$\pi^p = (1-\Delta)\pi(e) + \Delta\pi\left(p^C\right),$$

which ranges from $\pi(e) < \underline{\pi} = \tilde{\pi}^m(e)$ (for $T = +\infty$) to $(1-\delta)\pi(e) + \delta\pi\left(p^C\right)$ (for $T = 1$ and $x = 1$). Thus, as long as this upper bound exceeds $\tilde{\pi}^m(e)$,

[20] For any $p < e$:
$$\frac{\partial H}{\partial \delta}(p; e, \delta) = e\left[D(p+e) - D(2e)\right] > 0.$$

there exists $T \geq 1$ and $x \in [0,1]$ such that the penal code yields the minmax: $\pi^p = \tilde{\pi}^m(e) = \underline{\pi}$.

As $p^C$ satisfies (11), the final phase of this penal code (for $t > T+1$, and for $t = T+1$ with probability $x$) is sustainable. Furthermore, in the first $T+1$ periods the expected payoff increases over time (as the switch to $p^C$ comes closer), whereas the maximal profit from a deviation remains constant and equal to $\max_{p \leq e} pD(e+p) = \tilde{\pi}^m(e)$ (as $\tilde{p}^m(e) = r(e) < e$ for $e > \hat{p}$). Hence, to show that the penal code is sustainable it suffices to check that firms have no incentive to deviate in the first period, which is indeed the case if deviations are punished with the penal code:

$$\tilde{\pi}^m(e) = (1-\Delta)\pi(e) + \Delta\pi\left(p^C\right) \geq (1-\delta)\tilde{\pi}^m(e) + \delta\tilde{\pi}^m(e) = \tilde{\pi}^m(e).$$

There thus exists a penal code sustaining the minmax whenever the upper bound $(1-\delta)\pi(e) + \delta\pi\left(p^C\right)$ exceeds $\tilde{\pi}^m(e)$; as by construction $\pi\left(p^C\right) \geq \pi^N = \pi(\hat{p})$, this is in particular the case whenever

$$(1-\delta)\pi(e) + \delta\pi(\hat{p}) \geq \tilde{\pi}^m(e),$$

which amounts to $\delta \geq \underline{\delta}(e)$. Finally:

- $\underline{\delta}(e) \in (0,1)$ for any $e \in (\hat{p}, V)$, as then:

$$\pi(\hat{p}) = \max_p pD(\hat{p}+p) > \tilde{\pi}^m(e) = \max_p pD(e+p) > \pi(e) = eD(2e);$$

- $\underline{\delta}(V) = 0$, as $\tilde{\pi}^m(V) = \pi(V) = 0$, and

$$\lim_{e \longrightarrow \hat{p}} \frac{\tilde{\pi}^m(e) - \pi(e)}{\pi(\hat{p}) - \pi(e)} = \left.\frac{\frac{d\tilde{\pi}^m(e)}{de} - \frac{d\pi(e)}{de}}{-\frac{d\pi(e)}{de}}\right|_{e=\hat{p}} = \frac{D(2\hat{p}) + \hat{p}D'(2\hat{p})}{D(2\hat{p}) + 2\hat{p}D'(2\hat{p})} = 0,$$

  where the last equality stems from $\hat{p} = r(\hat{p}) = \arg\max_p pD(\hat{p}+p)$.

# F Proof of Proposition 3

i) We first show that, using reversal to Nash as punishment, firms can always sustain a stationary, symmetric equilibrium path in which they both charge constant price $p < \hat{p}$, for $p$ close enough to $\hat{p}$. This amounts to $\hat{K}(p; e, \delta) \geq 0$, where

$$\hat{K}(p; e, \delta) \equiv \pi(p) - (1-\delta)\pi^D(p; e) - \delta\pi(\hat{p}),$$

where

$$\pi^D(p; e) \equiv \max_{\tilde{p} \leq e} \tilde{p} D(p + \tilde{p}) = \begin{cases} r(p) D(p + r(p)) & \text{if } \; r(p) \leq e, \\ eD(p + e) & \text{if } \; r(p) > e. \end{cases}$$

Because $\pi^D(\hat{p}; e) = \pi(\hat{p})$, $\hat{K}(\hat{p}; e, \delta) = 0$ for any $e, \delta$. Furthermore:

$$\frac{\partial \hat{K}}{\partial p}(\hat{p}; e, \delta) = \pi'(\hat{p}) - (1-\delta)\hat{p}D'(2\hat{p}),$$

which using $\pi'(\hat{p}) = \hat{p}D'(2\hat{p})$, reduces to:

$$\frac{\partial \hat{K}}{\partial p}(\hat{p}; e, \delta) = \delta\hat{p}D'(2\hat{p}) < 0.$$

Hence, for $p$ close to $\hat{p}$, $\hat{K}(p; e, \delta) > 0$ for any $\delta \in (0, 1]$. If follows that cooperation on such price $p$ is always sustainable.

We now turn to perfect cooperation. Note first that it can be sustained by the minmax punishment $\underline{\pi} = \tilde{\pi}^m(e)$ whenever

$$\pi^m \geq (1-\delta)\pi^D(p^m; e) + \delta\tilde{\pi}^m(e),$$

or:

$$\delta \geq \overline{\delta}_1^N(e) \equiv \frac{\pi^D(p^m; e) - \pi^m}{\pi^D(p^m; e) - \tilde{\pi}^m(e)}.$$

Conversely, minmax punishments can be sustained using Abreu's optimal symmetric penal code whenever

$$(1-\delta)\pi(e) + \delta\pi^m \geq \tilde{\pi}^m(e), \tag{12}$$

or:

$$\delta \geq \overline{\delta}_2^N(e) \equiv \frac{\tilde{\pi}^m(e) - \pi(e)}{\pi^m - \pi(e)}.$$

Therefore, we can take $\overline{\delta}^N(e) \equiv \max\left\{\overline{\delta}_1^N(e), \overline{\delta}_2^N(e)\right\}$.

As $\overline{\delta}_1^N(\hat{p}) > \overline{\delta}_2^N(\hat{p}) = 0$ and $\overline{\delta}_1^N(V) > \underline{\delta}(V) = 0$, $\overline{\delta}^N(e) = \overline{\delta}_1^N(e) \geq \overline{\delta}_2^N(e)$ for $e$ close to $\hat{p}$ and for $e$ close to $V$. Furthermore, as $\tilde{\pi}^m(e)$ is continuous and coincides with $\pi(e)$ for $e = \hat{p}$, and $\pi^D(p^m; e) = eD(p^m + e)$ as long as $e < r(p^m)$ (where $r(p^m) > \hat{p}$), $\overline{\delta}_1^N(e)$ continuously prolongs the function $\overline{\delta}^N(e)$ defined in Proposition 2). Finally, both $\overline{\delta}_1^N(e)$ and $\overline{\delta}_2^N(e)$ lie below 1 (as $\tilde{\pi}^m(e) \leq \tilde{\pi}^m(\hat{p}) = \pi(\hat{p}) < \pi^m = \pi(p^m)$).

Next we show that

$$\overline{\delta}_1^N(e) = \frac{1}{1 + \dfrac{\pi^m - \tilde{\pi}^m(e)}{\pi^D(p^m; e) - \pi^m}}$$

decreases with $e$:

- For $e \geq r(p^m)$, $\pi^D(p^m; e) = r(p^m) D(p^m + r(p^m))$ does not vary with $e$ whereas $\tilde{\pi}^m(e) = \max_p pD(e + p)$ decreases with $e$; and so $\overline{\delta}_1^N(e)$ decreases with $e$.

- When $e \in [\hat{p}, r(p^m)]$, $\pi^D(p^m; e) = eD(p^m + e)$, and:

$$\begin{aligned}
\frac{d}{de}\left(\frac{\pi^m - \tilde{\pi}^m(e)}{eD(p^m + e) - \pi^m}\right) &= \frac{\begin{array}{c}[eD(p^m + e) - \pi^m][-r(e) D'(e + r(e))] \\ -[\pi^m - \tilde{\pi}^m(e)][D(p^m + e) + eD'(p^m + e)]\end{array}}{(eD(p^m + e) - \pi^m)^2} \\
&= \frac{\begin{array}{c}[eD(p^m + e) - \pi^m] D(e + r(e)) \\ +[\pi^m - \tilde{\pi}^m(e)][-(D(p^m + e) + eD'(p^m + e))]\end{array}}{(eD(p^m + e) - \pi^m)^2} \\
&> 0,
\end{aligned}$$

  where the second equality uses the first-order condition characterizing $r(e)$, and the inequality stems from all terms in the numerator being positive.

*ii)* As in the case of weak complementors, selling the incomplete technology cannot be more profitable than the static Nash:

$$\tilde{\pi}^m(e) = \max_p pD(e+p) < 2\pi^N = 2\pi(\hat{p}) = 2\max_p pD(\hat{p}+p).$$

Therefore, if collusion enhances profits ($\bar{v} > \pi^N$), there must exist some period $\tau \geq 0$ in which each firm $i$ charges a price $p_i^\tau$ not exceeding $e$, and the average price $\bar{p} = \frac{p_1^\tau + p_2^\tau}{2}$ moreover satisfies

$$\pi(\bar{p}) = \frac{\pi_1^\tau + \pi_2^\tau}{2} \geq \bar{v}.$$

To ensure that firm $i$ has no incentive to deviate, and for a given punishment payoff $\underline{v}$, we must have:

$$(1-\delta)\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta)\pi^D\left(p_j^\tau; e\right) + \delta\underline{v}.$$

Combining these conditions for the two firms yields:

$$(1-\delta)\frac{\pi^D(p_i^\tau; e) + \pi^D\left(p_j^\tau; e\right)}{2} + \delta\underline{v} \leq (1-\delta)\pi(\bar{p}) + \delta\frac{v_2^{\tau+1} + v_2^{\tau+1}}{2} \leq \pi(\bar{p}), \tag{13}$$

where the inequality stems from $\frac{v_1^{\tau+1}+v_2^{\tau+1}}{2} \leq \bar{v} \leq \pi(\bar{p})$. But the deviation profit $\pi^D(p; e)$ is convex in $p$ when $D'' \geq 0$,[21] and thus condition (13) implies $\underline{K}(\bar{p}; e, \delta, \underline{v}) \geq 0$, where

$$\underline{K}(p; e, \delta, \underline{v}) \equiv \pi(p) - (1-\delta)\max_{\tilde{p} \leq e} \tilde{p}D(p+\tilde{p}) - \delta\tilde{\pi}^m(e). \tag{14}$$

[21] In the range where $r(p) < e$, $\frac{\partial \pi^D}{\partial p}(p; e) = r(p)D'(p + r(p))$ and thus

$$\frac{\partial^2 \pi^D}{\partial p^2}(p; e) = r'D' + rD''(1+r') = -\frac{(D')^2}{2D' + rD''} > 0.$$

In the range where $r(p) > e$, $\frac{\partial \pi^D}{\partial p}(p; e) = eD'(p+e)$ and thus $\pi^D$ is convex if $D'' \geq 0$. Furthermore, the derivative of $\pi^D$ is continuous at $p = p_e \equiv r^{-1}(e)$:

$$\lim_{\substack{p \to p_e \\ p < p_e}} \frac{\partial \pi^D}{\partial p}(p; e) = \lim_{p \to p_e} eD'(p+e) = eD'(p_e + e) = \lim_{p \to p_e} r(p)D'(p + r(p)) = \lim_{\substack{p \to p_e \\ p > p_e}} \frac{\partial \pi^D}{\partial p}(p; e).$$

Conversely, if $\underline{K}(\bar{p}; e, \delta, \underline{v}) \geq 0$, then the stationary path $(\bar{p}, \bar{p})$ is an equilibrium path.

For any $\delta$, from Lemma 3 the minmax $\tilde{\pi}^m(e)$ can be used as punishment payoff for $e$ close to $\hat{p}$; the sustainability condition then amounts to $K(p; e, \delta) \geq 0$, where

$$K(p; e, \delta) \equiv \pi(p) - (1-\delta) \max_{\tilde{p} \leq e} \tilde{p} D(p + \tilde{p}) - \delta \tilde{\pi}^m(e).$$

Using $\tilde{\pi}^m(e) = \max_p pD(e+p)$ and noting that $\hat{p} = r(\hat{p}) < e$ implies $\pi(\hat{p}) = \max_p pD(\hat{p}+p) = \max_{p \leq e} pD(\hat{p}+p)$ for $\delta > 0$, we have:

$$K(\hat{p}; e, \delta) = \delta \left[ \max_p pD(\hat{p}+p) - \max_p pD(e+p) \right] > 0.$$

Furthermore, $K$ is concave in $p$ if $\pi^D(p; e)$ is convex in $p$, which is the case when $D'' \geq 0$. Thus, there exists $\underline{p}(e, \delta) \in [p^m, \hat{p})$ such that cooperation at price $p$ is feasible if and only if $\underline{p}(e, \delta) \leq p < \hat{p}$, and the set of sustainable Nash-dominating per-firm payoffs is then $[\pi(e), \overline{\pi}_1(e, \delta)]$, where $\overline{\pi}_1(e, \delta) \equiv \pi\left(\max\left\{p^m, \underline{p}(e, \delta)\right\}\right)$. Furthermore, using $\tilde{p}^m(e) = r(e) < \hat{p} < e$; we have, for $p < \hat{p} < e$:

$$\begin{aligned} \frac{\partial K}{\partial \delta}(p; e, \delta) &= \delta\left[\pi^D(p; e) - \tilde{\pi}^m(e)\right] \\ &= \delta\left[\max_{\tilde{p} \leq e} \tilde{p} D(p+\tilde{p}) - \max_{\tilde{p}} \tilde{p} D(e+\tilde{p})\right] > 0. \end{aligned}$$

Therefore, $\underline{p}(e, \delta)$ decreases with $\delta$, and thus $\overline{\pi}_1(e, \delta)$ weakly increases with $\delta$. Finally, note that $K(p; \hat{p}, \delta) = H(p; \hat{p}, \delta)$, where $H$ is defined by (10); hence the function $\overline{\pi}_1(e, \delta)$ defined here prolongs that of Proposition 2.

The function $\overline{\pi}_1(e, \delta)$ remains relevant as long as the minmax $\tilde{\pi}^m(e)$ is sustainable. When this is not the case, then $\underline{v}$ can be replaced with the lowest symmetric equilibrium payoff, which, using Abreu's optimal symmetric penal code, is of the form $(1-\delta)\pi(p^p) + \delta\pi(p^*)$, where $p^p$ is the highest price in $[\hat{p}, e]$ satisfying $\pi^D(p^p; e) - \pi(p^p) \leq \delta[\pi(p^*) - \pi(p^p)]$, and $p^*$ is the lowest price in $[p^m, \hat{p}]$ satisfying $\pi^D(p^*; e) - \pi(p^*) \leq \delta[\pi(p^*) - \pi(p^p)]$; we then have $\overline{\pi}_1(e,\delta) = \pi(p^*)$ and the monotonicity stems from $p^*$ and $p^p$ being respectively (weakly) decreasing and increasing with $\delta$.

# G Proof of Lemma 4

Suppose that $\min\left\{\frac{P^P}{2}, p_1^P, p_2^P\right\} \leq e$, and consider a period $t$, with individual licenses offered at prices $p_1^t$ and $p_2^t$. Let $\underline{p}_i^t = \min\left\{p_i^P, p_i^t\right\}$ denote the effective price for patent $i$, and $\underline{p}^t = \min\left\{\underline{p}_1^t, \underline{p}_2^t\right\}$ denote the lower one.

• Users buy the complete technology from the pool only if $P^P \leq \underline{p}^t + e$, which in turn implies $P^P \leq 2e$ (as $\underline{p}^t \leq \min\left\{p_1^P, p_2^P\right\}$, and by assumption, either $\frac{P^P}{2} \leq e$, or $\min\left\{p_1^P, p_2^P\right\} \leq e$); the industry profit is then $P^P D\left(P^P\right) \leq 2\pi^N = 2\pi(e)$, as the aggregate profit function $PD(P)$ is concave and maximal for $2p^m > 2e \geq P^P$.

• Users buy the complete technology by combining individual licenses only if $\underline{p}_i \leq e$ for $i = 1, 2$, in which case $\underline{p}_1 + \underline{p}_2 \leq 2e$ and the industry profit is $\left(\underline{p}_1 + \underline{p}_2\right) D\left(\underline{p}_1 + \underline{p}_2\right) \leq 2\pi^N$.

• Finally, users buy an incomplete version of the technology only if $\underline{p}^t + e \leq P^P$, which in turn implies $\underline{p}^t \leq e$ (as then $\underline{p}^t \leq \min\left\{p_1^P, p_2^P\right\}, P^P - e$, and by assumption, either $\min\left\{p_1^P, p_2^P\right\} \leq e$, or $P^P \leq 2e$); the industry profit is then $\underline{p}^t D\left(\underline{p}^t + e\right) \leq \left(\underline{p}^t + e\right) D\left(\underline{p}^t + e\right) \leq 2\pi^N$, as $\underline{p}^t + e \leq 2e$.

Therefore, the industry profit can never exceed the static Nash level.

# H Proof of Proposition 5

We have established that a pool price $p^P$ is stable if $L\left(p^P; e, \delta\right) \geq 0$, where

$$\begin{aligned} L(p; e, \delta) &\equiv \pi(p) - (1-\delta)\left[\pi(p) + (p-e)D(2p)\right] - \delta\pi(e) \\ &= \delta p D(2p) - (1-\delta)(p-e)D(2p) - \delta e D(2e). \end{aligned} \tag{15}$$

In particular, collusion on $p^m$ is feasible if $L(p^m; e, \delta) \geq 0$, or:

$$\delta \geq \bar{\delta}^P(e) = \frac{(p^m - e) D(2p^m)}{(p^m - e) D(2p^m) + \pi^m - \pi(e)} = \frac{1}{2 - \frac{e}{p^m - e}\frac{D(2e) - D(2p^m)}{D(2p^m)}},$$

where

$$\frac{d\overline{\delta}^P}{de}\left(e,\overline{\delta}^P(e)\right) = -\frac{\frac{\partial L}{\partial e}\left(p^m; e, \overline{\delta}^P(e)\right)}{\frac{\partial L}{\partial \delta}\left(p^m; e, \overline{\delta}^P(e)\right)}.$$

Clearly $\partial L/\partial\delta > 0$. Furthermore

$$\frac{\partial L}{\partial e}\left(p^m; e, \overline{\delta}^P(e)\right) = [1-\overline{\delta}^P(e)]D(2p^m) - \overline{\delta}^P(e)\pi'(e).$$

Using the fact that $L\left(p^m; e, \overline{\delta}^P(e)\right) = 0$,

$$\frac{\partial L}{\partial e}\left(p^m; e, \overline{\delta}^P(e)\right) \propto [\pi^m - \pi(e) - (p^m - e)\pi'(e)] < 0,$$

from the concavity of $\pi$. And so

$$\frac{d\overline{\delta}^P}{de} > 0.$$

More generally, sustaining a price $p^P \in (e, p^m]$ requires $\delta > 1/2$:

$$L(p; e, \delta) = (2\delta - 1)[pD(2p) - eD(2e)] + (1-\delta)e[D(2p) - D(2e)],$$

where the second term is negative and, in the first term, $\pi(p) > \pi(e)$. Note also that $L(e; e, \delta) = 0$ for all $e$, and that $L$ is concave in $p$ if $D'' \leq 0$.[22] The sustainability of collusion then hinges on $I(e)$ being positive, where

$$I(e,\delta) \equiv \frac{\partial L}{\partial p}(e; e, \delta) = (2\delta - 1)D(2e) + 2\delta e D'(2e).$$

We have:

$$\frac{\partial I}{\partial \delta}(e,\delta) = 2[D(2e) + eD'(2e)] > 0,$$

[22] As $pD(2p)$ is concave from Assumption A and $\delta > 1/2$, we have:

$$\frac{\partial^2 L}{\partial p^2} = (2\delta - 1)(pD(2p))'' + 4(1-\delta)eD''(2p) < 0.$$

where the inequality follows from $e < r(e)$ (as here $e < p^m$ $(< \hat{p})$); as

$$I(e, 1/2) = eD'(2e) < 0 < I(e, 1) = D(2e) + 2eD'(2e),$$

where the last inequality stems from $e < p^m$, then some collusion is feasible if $\delta$ is large enough, namely, $\delta \geq \underline{\delta}^P(e)$. Furthermore:

$$\frac{\partial I}{\partial e}(e, \delta) = 2(3\delta - 1)\left[D'(2e) + \frac{\delta}{3\delta - 1} 2eD''(2e)\right].$$

But $D'(2e) + 2eD''(2e) < 0$ from Assumption A and $\delta/(3\delta - 1) < 1$ from $\delta > 1/2$; and so

$$\frac{\partial I}{\partial e}(e, \delta) < 0,$$

implying that the threshold $\underline{\delta}^P(e)$ increases with $e$.

# I Proof of Proposition 9

From the above analysis, a pool (subject to independent licensing and unbundling) has no effect on the profit that can be achieved in case of entry, and thus has no effect on investment, in case of rivalry ($e \leq p^m$), as the pool then does not affect the scope for collusion. The "perfect screen" introduced in the previous section thus continues to make the pool welfare-neutral.

In case of complementors, allowing for a pool in case of entry has no effect either when the firms can already perfectly collude even without it (that is, when $\delta \geq \bar{\delta}^N(e)$), but otherwise allows the firms to lower prices, from some $p \in (p^m, p^N]$ down to $p^m$, and thus to increase profit, from $\pi(p)$ to $\pi^m$. Therefore, letting $I/(1-\delta)$ denote the investment cost:

- If $I > \pi^m$, there is no investment anyway, and thus the pool has again no impact.
- If $I < \pi(p)$, investment occurs both with and without the pool; allowing the pool however benefits users, whose surplus increase from $S(2p)$ to $S^m > S(2p)$.

- Finally, if $\pi(p) < I < \pi^m$, then the pool triggers entry, which benefits users whose surplus increase from $\tilde{S}^m$ (in the absence of entry) to $S^m$. Furthermore, despite some business stealing, the pool also increases total welfare; letting $W^m = W(p^m)$ and $\tilde{W}^m = \tilde{W}(\tilde{p}^m)$ denote total welfare with and without the pool, the impact of the pool satisfies:

$$\begin{aligned}
\Delta W^P &\equiv [W^m - I] - \tilde{W}^m \\
&= S^m + \pi^m - I - \tilde{W}^m \\
&> S^m - \tilde{W}^m \\
&= \int_0^{D(2p^m)} \left[V - p^m - F^{-1}(q)\right] dq - \int_0^{D(e+\tilde{p}^m(e))} \left[V - e - F^{-1}(q)\right] dq \\
&= \int_0^{D(e+\tilde{p}^m(e))} (e - p^m)\, dq + \int_{D(e+\tilde{p}^m(e))}^{D(2p^m)} \left[V - p^m - F^{-1}(q)\right] dq \\
&> 0,
\end{aligned}$$

where the first inequality stems from $\pi^m > I$ and the second one follows from $e > p^m$, $V - p^m > F^{-1}(q)$ for $q < D(2p^m)$, and $2p^m < e + \tilde{p}^m(e)$ (as $2p^m = r(0)$ and $\tilde{p}^m(e) = r(e)$, where $r'(.) > -1$).

# J Proof of Proposition 12

Assume that $e < p^m$, which implies $p^N = e$, and consider a symmetric equilibrium in which all pool members charge the same effective price: $\min\left\{p_i^P, p_i\right\} = p^*$ for $i = 1, ..., n$. From Lemma 5, to generate higher profits than the static Nash outcome, users must buy $m^* < n$ patents, and so this price must satisfy $p^* > e$; members' equilibrium profit is then:

$$\pi^* = \frac{m^*}{n} p^* D\left(m^* p^* + V(n) - V(m^*)\right).$$

The price $p^*$ can be sustained by reversal to Nash if and only if:

$$\pi^* \geq (1 - \delta)\, \pi^D(p^*) + \delta \pi^N,$$

where $\pi^N = eD(ne)$ denotes the static Nash profit and $\pi^D(p^*)$ denotes the most profitable deviation from $p^*$, subject to charging a price $p^D \leq p_i^P$. But as the deviating price must lie below $p^*$ (otherwise, the member's patent would be excluded from users' basket), it is not constrained by the pool price $p_i^P$; therefore, the deviation cannot be less profitable than in an alternative candidate equilibrium in which, in the absence of the pool, all members would charge $p^*$. Hence, the pool cannot sustain higher symmetric prices than what the firms could sustain in a symmetric equilibrium in the absence of the pool.

# K Proof of Lemma 5

To prove Lemma 5, we first show that selling all $n$ products requires charging an average price weakly lower than $e$:

**Lemma 6** *Selling all $n$ products requires charging a total price $P \leq ne$.*

**Proof.** Let $\hat{m} = \arg\max_{m<n}\{V(m) - me\}$ and consider a given price profile $(p_1, ..., p_n)$; for $k = 1, ..., n$, let $p_{(k)}$ denote the $k^{th}$ lowest price, and $P_m = \sum_{k=1}^{m} p_{(k)}$ denote the sum of the $m$ lowest prices (with the convention $P_0 = 0$). Selling $n$ rather than $\hat{m}$ products requires:

$$V(n) - P_n \geq V(\hat{m}) - P_{\hat{m}},$$

or, using $V(n) - ne = V(\hat{m}) - \hat{m}e$:

$$\sum_{k=\hat{m}+1}^{n} p_{(k)} \leq (n - \hat{m})\, e.$$

This in turn implies, for $k \leq \hat{m}$:

$$p_{(k)} \leq p_{(\hat{m}+1)} \leq \frac{\sum_{k=\hat{m}+1}^{n} p_{(k)}}{n - \hat{m}} \leq e,$$

and thus:

$$P_n = \sum_{k=1}^{\hat{m}} p_{(k)} + \sum_{k=\hat{m}+1}^{n} p_{(k)} \leq \hat{m}e + (n - \hat{m})\, e = ne.$$

■

To conclude the proof of Lemma 5, assume now $e < p^m$, and suppose that a price profile $(p_1, ..., p_n)$ induces users to buy all $n$ products. The aggregate profit is then $\Pi(P) = PD(P)$, where $P = \sum_{k=1}^{n} p_k$ denotes the total price. But this profit function is concave in $P$ under Assumption A, and thus increases with $P$ in the range $P \leq P^m$, where $P^m = np^m > ne$. As selling all $n$ products require $P \leq ne$ from the above Lemma, the aggregate profit thus cannot exceed that of the static Nash, $neD(ne)$.